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EDITORIAL

The Mechanism of Magnetic Attraction

TO calculate the mechanical force on an element of current-carrying conductor in a magnetic field, one multiplies the current by the length and by the component of the magnetic induction normal to the length. There must be an equal and opposite mechanical reaction on the source of the magnetic field. If the resultant magnetic field is made up of several components due to different sources they can each be considered separately in order to determine the reactions on the different sources. The element of conductor may be replaced by a moving electric charge.

We now propose to apply these elementary principles to the case of a piece of iron in the field of a permanent magnet. Every atom of the iron consists of a stationary nucleus in which we are not interested, and a number of revolving electrons. The whole mass of iron may be pictured as relatively empty space in which myriads of electrons revolve in planes which are controlled by the joint action of neighbouring atoms and the externally applied field. Each electron may be regarded as a small current-carrying coil situated in a magnetic field which is the resultant of two component fields, one due to all the other revolving electrons in the mass of iron, and the other due to the neighbouring magnet.

It is interesting to consider the nature of the forces that can act on the electron. If the magnetic field is uniform and normal to the plane of rotation, the force can only tend to increase or decrease its radius of rotation; if the field is uniform and in the plane of rotation, the force can only tend to rotate the plane in which the electron revolves. Translational forces can only be caused by non-uniformity of the magnetic field. These considerations apply either to the resultant or to the component fields. If one could integrate the forces acting on all the electrons in the mass of iron due to the resultant magnetic field one would obtain the resultant force acting on the mass, but it is much more enlightening to picture the integration carried out separately for the two component magnetic fields. The integration of all the forces due to the field produced by the rotating electrons in the mass of iron must be zero, since every elemental force has an equal and opposite reaction on some other part of the mass of iron. If the magnet could be removed without any change in the electronic structure of the iron, the integrated force would remain unchanged and it must obviously be zero in this case since the piece of iron, which is now, in effect, a permanent magnet, could not tend to move or rotate. There will, of course, be stresses within the iron but no externally applied force tending to move it. We are neglecting the effect of

the earth, which is, of course, an externally applied magnet.

We are left, therefore, with the other term in the integration, namely the force on the iron due to the rotation of its electrons in the magnetic field of the magnet *unaffected by the presence of the iron*. We are neglecting any small effect that the presence of the piece of iron may have on the electronic orientations within the magnet. The point that we wish to emphasize is that, if we are not concerned with internal forces within the mass of iron, but only with the external forces tending to move it, then we are not interested in the resultant magnetic field but only in the field as it would be if the piece of iron were not there.

This throws light upon a point which has probably worried many who have tried to obtain a clear physical picture of what is happening in the iron.

Fig. 1 (a) shows the north pole of, say, a cylindrical bar magnet and a piece of iron

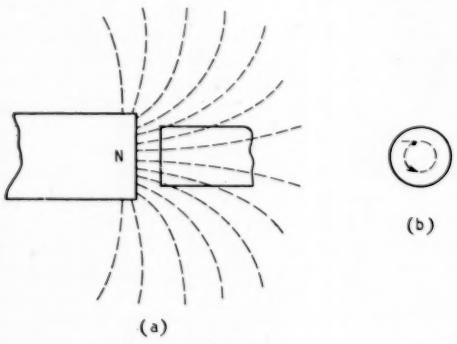


Fig. 1.

placed near it. In Fig. 1 (b) we are supposed to be situated on the pole and looking at the face of the piece of iron. The magnetic flux is going from us into the iron and the result of the magnetization is an orientation to some extent of the electron orbits in the direction to increase the flux; a typical electron orbit is shown, greatly magnified, in the figure. Now in order to exert a force of attraction or replusion on this electron there must be a magnetic field in the plane of the paper and a little consideration shows that if the field were radially outwards, the electron, and consequently the mass of iron, would be attracted, whereas if the field were radially inwards the iron would be repelled. The fact that the iron is attracted shows that the field causing the attraction diverges on passing into the iron which is true of the original magnet field shown in Fig. 1 (a) although the resultant field converges on entering the iron.

In Fig. 2 a horse-shoe magnet is attracting an iron plate; the magnetic field shown is that due to the magnet alone in the absence of the iron plate. The electron orbits will be orientated to some extent toward planes normal to the field as indicated in Fig. 2, and the forces on the electrons will be of a complex character, made up of forces in the general direction of the field, as in Fig. 1, and of forces at right angles to the field, but acting in both cases in the direction in which the flux through the orbit will be increased. At a point such as P the electron is urged in the direction shown because one side of its orbit is in a stronger field than the other side and there is consequently a resultant force on it. At a point such as Q both types of force will be operative, or at least, a resultant force which may be regarded as made up of two such components.

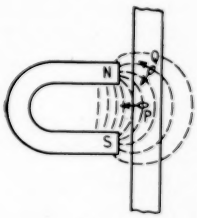


Fig. 2.

This method of approach brings out very clearly that the attractive force on the iron is not exerted on any surface polarity but acts throughout the mass, and that the force acting at any point does not depend on the resultant magnetic induction at the point but on the rate at which the magnetizing force due to the magnet alone is changing either longitudinally or laterally. G. W. O. H.

THE TRAVELLING-WAVE TUBE*

Centimetre-Wave Amplifier

By R. Kompfner

SUMMARY.—A theory of a tube based on the principle of interaction between travelling waves and electrons is developed with the aid of a number of simplifying assumptions. To begin with, the modulation of the beam by the wave is considered ignoring any reaction of such modulation on the wave. Similarly, the wave produced by a modulated beam is investigated. Next, the influence of the modulation of a beam, which has been produced by a wave, on that wave, is calculated and this process is carried on for an infinite number of actions and reactions. The sum of all these actions, representing the complete interaction between electrons and wave, is derived for the case of wave and electrons travelling at the same velocity. Expressions for the amplification and the noise factor of a tube under these conditions are given.

A description is then given of experiments on a coaxial line with a helical inner conductor, which has been chosen as a structure for slowing down the wave to velocities of practicable electron beams. The interaction of beam and wave is investigated and is found to accord substantially with theory. Tubes are described which are run as amplifiers and the results obtained are given.

1. Introduction

PROGRESS in the technique of radio has for some time been intimately connected with a tendency to employ ever shorter wavelengths. This necessarily led to ever smaller circuit components and spacings in valves, until a point was reached where devices based on conventional conceptions ceased to function efficiently and eventually to function at all. A very important step forward was made with valves of the drift-tube or klystron type, for they succeeded by turning a very serious difficulty of the conventional valve into an advantageous and, indeed, an essential feature. The difficulty lay in the fact that the transit time of electrons in the inter-electrode space becomes considerable in comparison with the time of one cycle of the operating frequency. Together with the use of non-radiating, highly-resonant structures for circuits the utilization of the transit time in the drift-tube has enabled it to perform very well at frequencies at which the conventional triode, tetrode, etc., can only be made to work under difficulties involving the use of microscopically fine wire-mesh grids and spacings of extremely small clearances and tolerances. At these wavelengths the drift-tube has still a very macroscopic and robust structure and is easily built and handled.

However, as the wavelength is still further reduced transit-time reappears as difficulty, even with drift-tubes. In all valves, particularly in drift-tubes, the energy interaction between electron stream and electromagnetic field is very much localized. There is thus a limit to the frequencies which can be reached even by drift-tubes and that is again given by transit-time, just as in the case of the conventional valve. It is a natural and logical step to seek an alternative to interaction by means of intense localized fields; such an alternative is given by the interaction between an electron stream and a *travelling* electromagnetic field. Obviously, wave and electrons must travel at about the same velocity, so that particular electrons will stay in the same phase of the field for a considerable time and so experience a cumulative effect. Thus, for instance, some electrons will be accelerated all the time while they travel with the corresponding portion of the field and others will be retarded all the time. This makes up for the fact that we cannot expect travelling fields to have intensities as high as those of localized and stationary fields. Stationary fields, or standing waves as they can be termed, are usually characterized by high intensities; however, electrons spend very little time in them, and it will be shown later that the energy transfer between travelling field and electrons can be made considerably more effective than the energy transfer between electrons and a localized stationary field.

For reasonable electron-beam voltages (reasonable being more than a few hundred and less than a few thousand volts) it turns out that the wave is required to travel at velocities around one-tenth of that of light. This rules out at once the use of ordinary

* MS. accepted by the Editor, October, 1946.

waveguides or transmission lines of the coaxial type with cylindrical symmetry. The former have indeed phase velocities higher than that of light, while, in addition, the fundamental mode in the latter offers only transverse electric fields, which are not so desirable as axial fields from several points of view.

A loaded transmission line, consisting of an inner conductor in the form of a helix concentric with a solid metallic outer conductor, has been found to be very suitable for slowing down the wave to the desired value, and has been successfully used in the experiments to be described later on. At this stage it is appropriate to mention that both transverse and axial fields do exist in such a transmission line, the latter mainly within the helix, and the former mainly between the helix and the outer conductor. Thus, the assumption of periodic electric fields travelling with velocities of the order of one-tenth of that of light is physically realizable and is made the basis of the following calculations where the results of the interaction between electrons and travelling wave are estimated.

2. Assumptions and Definitions

In this section a theory of interaction between an electron beam and a travelling wave will be outlined, subject to a number of rather drastically simplifying assumptions. The assumptions are :—

(*a*) that it is admissible to calculate the effect of the wave on the beam as if it were isolated ; i.e., ignoring any reaction of the beam on the wave.

(*b*) that it is similarly admissible to calculate the effect of the modulation of a beam on the wave, ignoring the effect of the wave on the beam.

(*c*) that any displacements of electrons from their undisturbed positions are very small. This is equivalent to postulating small-signal conditions.

(*d*) that attenuation of the wave due to losses in conducting surfaces, and in any dielectric, is neglected.

(*e*) that space-charge repulsion effects are neglected.

(*f*) that we are dealing with a very thin electron beam composed of electrons of uniform initial velocity.

A wave is characterized by the peak voltage V, its wavelength λ and a constant velocity of propagation u. V is related to the mean signal power W by

$$V = \sqrt{2WZ_0} \text{ volts} \quad .. \quad .. \quad (1)$$

where Z_0 is the characteristic impedance of the transmission line along which the wave is travelling. The instantaneous voltage at any point x in the line and at the time t is described by

$$V(xt) = V \begin{matrix}\sin \\ \cos\end{matrix}\left\{\omega t - \frac{2\pi x}{\lambda}\right\} \text{volts} \quad .. \quad (2)$$

where ω is the angular frequency of the wave.

An electron beam is shot through the line in the direction of travel of the wave. The beam current is denoted by I_0 and its uniform velocity v is given by the relation

$$v = \sqrt{\frac{2e}{m}}\, V_0 \text{ cm/sec} \quad .. \quad .. \quad (3)$$

where V_0 is the accelerating potential.

The field at the point of the cross-section of the line where the electron beam is situated is resolved into a *transverse* field ϕ_T (or radial, if the line is rotationally symmetrical) and an *axial* field ϕ_A.

Now in a planar line

$$\phi_T = \tau \frac{V}{D} = \frac{\tau}{D} V \sin\left(\omega t - \frac{2\pi x}{\lambda}\right) \text{volts/cm} \quad .. \quad (4)$$

where D is the distance between the conductor on which the voltage V is supposed to exist and the other conductor which is supposed to be at earth potential. τ is a numerial factor, less than unity, indicating how much weaker the field is at the point of the electron beam than it would be if λ were very large compared with D. τ is determined by the geometry of the line and by the kind of wave propagated in the line. The planar configuration has been chosen in order to simplify the expressions. There is, however, no principal difficulty in the way of choosing any other type of configuration.

Similarly, the axial field ϕ_A is given by

$$\phi_A = \alpha \frac{\partial V}{\partial x} = \frac{2\pi\alpha}{\lambda} V \sin\left(\omega t - \frac{2\pi x}{\lambda}\right) \text{volts/cm} \quad (5)$$

For the sake of convenience, a field which accelerates an electron in the positive direction has been considered a positive field. Further Equ. (5) presupposes that the voltage giving rise to the field ϕ_A is of the form $V \cos\left(\omega t - \frac{2\pi x}{\lambda}\right)$.

3. First Order Beam Displacement

If t_0 is the time of entry into the line of an electron, then the force on it at time t is given by

$$m\frac{d^2s}{dt^2} = e\phi \sin\left[\omega t_0 + \frac{2\pi}{\lambda}(u-v)(t-t_0)\right] \quad .. \quad .. \quad .. \quad .. \quad (6)$$

where ϕ is the amplitude of the field, in both the axial or transverse case; $(u-v)(t-t_0)$ is the longitudinal or axial shift of an electron relative to the wave; $v(t-t_0) = x$ the distance travelled; and s is the distance from the undisturbed position.

$\frac{d^2s}{dt^2}$ integrated gives

$$\frac{ds}{dt} = -\frac{e\phi\lambda}{2\pi m(u-v)}\left\{\cos\left[\omega t_0 + \frac{2\pi}{\lambda}(u-v)(t-t_0)\right] - \cos\omega t_0\right\}$$

$$= \frac{e\phi x}{mv}\frac{\sin\theta/2}{\theta/2}\cdot\sin(\omega t_0 + \theta/2) \quad .. \quad (7)$$

where $\theta = \frac{2\pi x}{\lambda}\left(\frac{u}{v} - 1\right)$ is the transit angle of an electron relative to the wave.

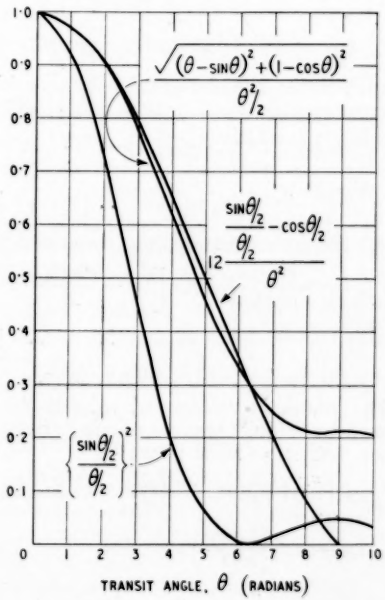


Fig. 1. *Values of three expressions as functions of the transit angle.*

Another integration gives the displacement

$$s = \frac{e\phi x^2}{mv^2}\left[(\sin\theta-\theta)^2 + (1-\cos\theta)^2\right]^{\frac{1}{2}}\theta^{-2} \sin\left[\omega t_0 + \tan^{-1}\frac{\theta-\sin\theta}{1-\cos\theta}\right] \quad (8)$$

The amplitude of Equ. (8) is plotted in Fig. 1.

When $u = v$, $\theta \to 0$ and

$$s = \frac{e\phi x^2}{2mv^2}\sin\omega t_0 = \frac{\tau V x^2}{4V_0 D}\sin\omega t_0 \quad (9)$$

in the transverse field case, which is identical with the deflection obtained within a pair of parallel plates of length x when the frequency is very low.

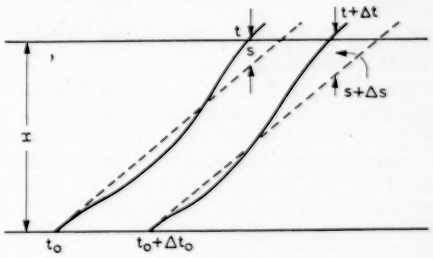


Fig. 2. *The space-time diagram of the electron flow in the valve.*

4. Current distribution in Beam

The effect of the *axial* displacement of the beam on its current distribution can be treated as follows:—

By the principle of conversation of charge it follows that:

$$\Delta t_0 \,.\, I_0 = \Delta t \,.\, I_x$$

$$\text{or } I_x = I_0\frac{\partial t_0}{\partial t} \quad .. \quad .. \quad .. \quad (10)$$

where Δt_0 is the small time interval taken by a particular amount of charge to cross the plane $x = 0$; I_0 is in the initially uniform beam current; Δt is the time interval taken by the same amount of charge to cross the plane x; and I_x is the beam current at x.

From the "space-time" diagram, Fig. 2, we also have

$$[t - t_0]v + s = x \quad .. \quad .. \quad .. \quad (11)$$

$$[(t+\Delta t) - (t_0+\Delta t_0)]v + (s+\Delta s) = x \quad (12)$$

Hence $(\Delta t - \Delta t_0)v + \Delta s = 0$

$$\text{or } \frac{\partial t_0}{\partial t_{(x)}} = 1 + \frac{1}{v}\frac{\partial s}{\partial t_{(x)}} \quad .. \quad .. \quad (13)$$

and $I_x = I_0\left(1 + \frac{1}{v}\frac{\partial s}{\partial t_{(x)}}\right)$ .. .. (14)

In the special case when s is given by Equ. (8)

$$I = I_0\left\{1 + \frac{\omega}{v}\frac{e\phi x^2}{mv^2}\left[(\sin\theta - \theta)^2 + (1-\cos\theta)^2\right]^{\frac{1}{2}}\theta^{-2}\cos\left[\omega t_0 + \tan^{-1}\frac{\theta - \sin\theta}{1-\cos\theta}\right]\right\}$$
.. .. .. (15)

That is the current at x due to a voltage wave of the form

$$+ V\cos\left(\omega t - \frac{2\pi x}{\lambda}\right)$$

We recall that ϕ was given by $|\phi_A| = \frac{2\pi\alpha V}{\lambda}$

5. Voltage Induced by Transverse Displacement Modulation

In order to determine the voltage induced by a transverse displacement modulation of the beam we consider a beam modulated in the way shown in Fig. 3; Λ is the wavelength in the beam as determined by the frequency and the beam velocity in accordance with the relation

$$\frac{\omega}{v} = \frac{2\pi}{\Lambda}$$

This will be termed "displacement modulation."

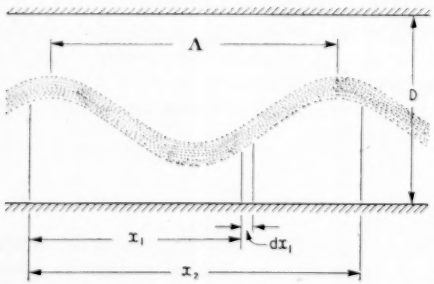


Fig. 3. *A transverse deflection modulated beam of electrons is illustrated.*

Let $s(xt)$ denote the transverse displacement of the beam. Considering only an element of length dx_1 at x_1, the amount of charge contained within that length, denoted by dq, is given by

$$dq = \frac{I_0}{v}.dx_1 \quad .. \quad .. \quad .. \quad (16)$$

and this elemental charge is moving to and fro across the line with an apparent transverse velocity v' given by

$$v' = \frac{\partial s(x_1 t)}{\partial t} \quad .. \quad .. \quad .. \quad (17)$$

The elemental current induced in the line is

$$di_1 = -\frac{\tau\, dq\,.\,v'}{D} = -\frac{\tau I_0}{vD}.\frac{\partial s(x_1 t)}{\partial t} \quad .. \quad (18)$$

one half of which will flow to the right (the forward direction) and one half to the left (backwards) producing an element of voltage at x_1

$$dV_1 = \tfrac{1}{2}\,di_1\,.\,Z_0 = -\frac{\tau I_0 Z_0}{2vD}\frac{\partial s(x_1 t)}{\partial t}.dx_1 \quad (19)$$

This elemental voltage wave will be propagated both in the forward and backward directions with the velocity u and to obtain the complete voltage at a point x_2 we have to sum, between $x_1 = 0$ and $x_1 = x_2$, all the elemental voltages which are travelling forward, and between $x_1 = x_2$ and $x_1 = l$ the elemental voltages travelling backwards.

By way of illustration, let us assume that

$$s(x_1 t) = s_0 \sin\left(\omega t - \frac{2\pi x}{\Lambda}\right) \quad .. \quad (20)$$

which is a uniform displacement modulation of amplitude s_0.

We have

$$\frac{\partial s(x_1 t)}{\partial t} = \omega s_0 \cos\left(\omega t - \frac{2\pi x_1}{\Lambda}\right) \quad .. \quad (21)$$

and

$$dV_1 = -\frac{\tau\omega I_0 Z_0 s_0}{2vD}\cos\left(\omega t - \frac{2\pi x_1}{\Lambda}\right)dx_1 \quad (22)$$

At this point x_2 the elemental voltage, taking the ensuing phase-shift into account, will be

$$dV_{12} = -\frac{\tau\omega I_0 Z_0 s_0}{2vD}\cos\left[\omega t - \frac{2\pi x_1}{\Lambda} - \frac{2\pi(x_2 - x_1)}{\lambda}\right]dx_1 \quad .. \quad .. \quad (23)$$

due to the forward-travelling wave and

$$dV^*_{12} = -\frac{\tau\omega I_0 Z_0 s_0}{2vD}\cos\left[\omega t - \frac{2\pi x_1}{\Lambda} + \frac{2\pi(x_1 - x_2)}{\lambda}\right]dx_1 \quad .. \quad .. \quad .. \quad (24)$$

due to the backward-travelling wave.

The complete voltage at x_2 due to all forward-travelling waves excited between $x_1 = 0$ and $x_1 = x_2$ is given by

$$V_2 = \int_{x_1=0}^{x_1=x_2} dV_{12} = -\frac{\pi\tau I_0 Z_0 s_0 x_2}{D\Lambda}\frac{\sin\theta/2}{\theta/2}\cos\left(\omega t - \pi x_2\frac{\Lambda+\lambda}{\Lambda\lambda}\right) \quad .. \quad .. \quad .. \quad (25)$$

where $\theta = 2\pi x_2\frac{\Lambda+\lambda}{\Lambda\lambda} = \frac{2\pi x_2}{\lambda}\left(\frac{u-1}{v}\right)$

as before.

The amplitude of (25) is plotted in Fig. 1.

When $u \to v$, $\theta \to 0$ and (25) reduces to

$$V_2 = -\frac{\pi\tau I_0 Z_0 s_0}{\Lambda\lambda} x_2 \cos\left(\omega t - \frac{2\pi x_2}{\lambda}\right) \quad (26)$$

Equ. (25) represents a wave travelling with a velocity $\frac{2uv}{u+v}$ and of varying amplitude, while (26) represents a wave travelling with the velocity u and uniformly increasing amplitude.

The voltage at x_2 due to backward-travelling waves denoted by V_2^* can be calculated in a similar manner; if the total length of the transmission line is l we have

$$V_2^* = -\frac{\pi\tau I_0 Z_0 s_0 (l - x_2)}{D\Lambda} \frac{\sin\phi}{\phi} \cos\left\{\omega t - \pi l \frac{\Lambda + \lambda}{\Lambda\lambda} + \pi x_2 \frac{\Lambda - \lambda}{\Lambda\lambda}\right\} \quad (27)$$

where $\phi = \pi(l - x_2)\frac{\Lambda + \lambda}{\Lambda\lambda}$.

When $u = v$, Equ. (27) reduces to

$$V_2^* = -\frac{\tau I_0 Z_0 s_0}{2D} \sin\left[\frac{2\pi}{\lambda}(l - x_2)\right] \cos\left(\omega t - \frac{2\pi l}{\lambda}\right) \quad .. \quad .. \quad .. \quad (28)$$

Equ. (27) represents a wave of varying amplitude travelling backwards at a rate given by $\frac{2uv}{u-v}$, while (28) represents a kind of standing wave, its amplitude varying with distance. No energy is being passed along.

The amplitude of (28) will be less than that of (26) provided

$$\frac{x_2}{\lambda} > \frac{2}{\pi}$$

Hence this wave will be of little importance for tubes many wavelengths long.

6. Voltage Induced by Density Modulation

Let the density-modulation of the beam be $I = I(xt)$.

An element of charge dq at x_1, dx_1 long, will then be given by

$$dq = \frac{I}{v} dx_1 = \frac{I(x_1 t)}{v} dx_1 \quad .. \quad .. \quad (29)$$

If dV_1 is the potential across the line due to the presence of dq then by definition (Equ. 5) the potential at the place of the electron beam is $\alpha \,.\, dV_1$. dV_1 will be proportional to dq.

Hence we write

$$dV_1 = -p \,.\, dq \quad .. \quad .. \quad .. \quad (30)$$

where p is some constant of proportionality.

The energy of the element of charge dq at x_1 is

$$\tfrac{1}{2}\, dq \,.\, \alpha \,.\, dV_1 \quad .. \quad .. \quad .. \quad (31)$$

and the rate of change of energy with time (that is, the power flowing into the line) is

$$2dW = \tfrac{1}{2}\alpha \frac{\partial(dq \,.\, dV_1)}{\partial t} \quad .. \quad .. \quad (32)$$

We write $2dW$ because $1dW$ can be imagined to flow away to the right with $1dW$ flowing to the left.

$$\text{But } dW = \frac{(dV_1)^2}{Z_0} \quad .. \quad .. \quad .. \quad (33)$$

$$\text{Hence } \frac{\alpha}{2}\frac{\partial(dq \,.\, dV_1)}{\partial t} = 2\frac{(dV_1)^2}{Z_0} \quad .. \quad (34)$$

$$\text{But } dq \,.\, dV_1 = -p \,.\, (dq)^2$$

$$\text{and } \frac{\partial(dq \,.\, dV_1)}{\partial t} = -2p \,.\, dq \frac{\partial(dq)}{\partial t} = -2dV_1 \frac{\partial(dq)}{\partial t} \quad .. \quad .. \quad .. \quad (35)$$

which gives, when inserted in (34)

$$dV_1 = -\frac{\alpha Z_0}{2} \cdot \frac{\partial(dq)}{\partial t} = -\frac{\alpha Z_0}{2v}\frac{\partial I(x_1 t)}{\partial t} dx_1 \quad .. \quad .. \quad .. \quad (36)$$

Equ. (36) is analogous to (19), and all the subsequent steps in Section 5 have to be repeated in this section.

Let the density modulation of the beam be of the form

$$I(x_1 t) = aI_0 \sin\left(\omega t - \frac{2\pi x_1}{\Lambda}\right) \quad .. \quad (37)$$

where a is a dimensionless number representing the depth of modulation, then

$$\frac{\partial I(x_1 t)}{\partial t} = \omega a I_0 \cos\left(\omega t - \frac{2\pi x_1}{\Lambda}\right) dx_1 \quad (38)$$

and

$$dV_1 = -\frac{\alpha\omega a I_0 Z_0}{2v} \cos\left(\omega t - \frac{2\pi x_1}{\Lambda}\right) dx_1 \quad (39)$$

which is similar to Equ. (22) except that a replaces τ and S_0/D is exchanged for a.

Hence the voltage at x_2 due to forward-travelling waves is

$$V_2 = -\frac{\pi\alpha a I_0 Z_0 x_2}{\Lambda} \frac{\sin\theta/2}{\theta/2} \cos\left(\omega t - \pi x_2 \frac{\Lambda + \lambda}{\Lambda\lambda}\right) \quad .. \quad (40)$$

and that due to backward-travelling waves

$$V_2^* = -\frac{\pi\alpha a I_0 Z_0 (l - x_2)}{\Lambda} \frac{\sin\phi}{\phi} \cos\left\{\omega t - \pi l \frac{\Lambda + \lambda}{\Lambda\lambda} + \pi x_2 \frac{\Lambda - \lambda}{\Lambda\lambda}\right\} \quad (41)$$

7. Noise-Voltage Induced by Shot Fluctuation

The general cases treated by way of examples in Sections 5 and 6 are useful in describing the manner in which shot noise enters into the line.

We will treat the case of transverse interaction first. Consider a beam of thickness d; a shot-noise voltage will be induced into the line by virtue of the random fluctuations of charge density across the beam. It is sufficient to treat the low-frequency case of a pair of deflecting plates, D distant from each other, through which a beam of mean current I_0/cm width is shot. Let the beam be of infinite width in the direction perpendicular to the direction of travel, so that the problem becomes one-dimensional, as shown in Fig. 4.

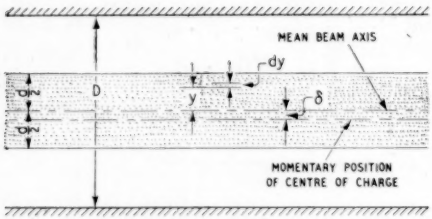


Fig. 4. *Random charge density distribution in the electron beam.*

We imagine all the charge between the plates to be concentrated in a plane, the so-called centre of charge plane, which will move to and fro between the plates in accordance with the momentary charge distribution. In other words, the centre of gravity of the electron cloud will fluctuate, and once we know the rate at which it moves we can apply Equ. (25), which will give us the voltage induced in the line.

The current in the thin layer dy at y cm from the mean beam axis is $\frac{dy}{d} I_0$ where I_0 is the current in the whole beam.

Suppose there is a momentary excess i in that layer. That will entail a shift δ of the position of the centre of charge away from the beam axis. By elementary mechanical analogy we have

$$\delta \cdot I_0 = y \cdot i$$

or taking mean squares .. (42)

$$\overline{\delta^2} = \overline{i^2} \cdot \frac{y^2}{\overline{I_0}^2} \quad .. \quad .. \quad .. \quad (43)$$

$$\text{But } \overline{i^2} = 2e\frac{dy}{d} I_0 df \Gamma^2 \quad .. \quad .. \quad .. \quad (44)$$

where Γ^2 is the usual space-charge smoothing factor.

By the theorem of the linear addition of the mean squares of fluctuations we find the resultant mean-square displacements of the position of the centre of charge; denoted by $\overline{\Delta^2}$ as follows:

$$\overline{\Delta^2} = \int_{-d/2}^{+d/2} \overline{\delta^2} = \frac{2edf\Gamma^2}{I_0 d} \int_{-d/2}^{+d/2} y^2 dy$$

$$= \frac{2edf\Gamma^2 d^2}{12\, I_0} \quad .. \quad .. \quad (45)$$

For a very narrow band of frequencies we can write

$$s(x_1 t) = \overline{\Delta} \sin\left(\omega t - \frac{2\pi x_1}{\lambda}\right) \quad .. \quad (46)$$

or identify Δ with s_0 and proceed according to Equ. (25). Thus the mean-squared shot voltage denoted by $\overline{V_2^2}$ is found to be

$$\overline{V_2^2} = \frac{\pi^2 \tau^2 Z_0^2}{12} \frac{2eI_0 df\Gamma^2}{\Lambda^2 D^2} x_2^2 d^2 \frac{\sin^2(\theta/2)}{(\theta/2)^2} \quad (47)$$

The case of the interaction of the shot fluctuations with the axial fields is treated as follows:—

The density modulation of the electron beam has been described by Equ. (37) where I_0 is the amplitude of the a.c. component of the beam current. In the case of shot-noise, for a very narrow band of frequencies, we put

$$\overline{a^2} I_a^2 = 2eI_0\, df \,.\, \Gamma^2 \quad .. \quad .. \quad (48)$$

$$\text{hence } \overline{a^2} = \frac{2e\, df\Gamma^2}{I_0} \quad .. \quad .. \quad (49)$$

This gives, in conjunction with Equ. (40) an expression for the mean-square induced shot-voltage, again denoted by $\overline{V_2^2}$ as follows:

$$\overline{V_2^2} = \frac{\pi^2 \alpha^2 Z_0^2}{\Lambda^2} \frac{2eI_0\, df\Gamma^2}{} x^2 \cdot \frac{\sin^2(\theta/2)^2}{(\theta/2)^2}$$

.. .. .. (50)

8.—Voltage Induced by a Signal-Modulated Beam

So far we have treated the interaction between wave and beam as if it were purely a case of either action of wave on the beam, or of action of the beam on the wave. Thus we are not strictly justified in talking of *inter*-action, up to now. In this and the following sections, however, we are going a step further; we will enquire into what happens when a density modulation which itself has been produced in a beam by a wave induces a secondary wave in the line. This will be done for the general case of beam and wave not having the same velocity.

The special case of the velocities being equal (i.e., $u = v$) can be derived from the general case and can be used in deriving the beam-density modulation due to that secondary wave, and again, of the tertiary wave induced in the line by that density modulation, and so on. Thus, the general scheme of higher-order waves is developed and eventually summed for an infinite number of higher-order waves. This sum of all the waves can truly be said to represent the result of *interaction* between wave and beam.

Let the initial voltage be

$$V\cos\left(\omega t-\frac{2\pi x}{\lambda}\right) \quad .. \quad .. \quad .. \quad (51)$$

The axial field will be

$$\phi=\frac{2\pi\alpha V}{\lambda}\sin\left(\omega t-\frac{2\pi x}{\lambda}\right)=\phi_A\sin\left(\omega t-\frac{2\pi x}{\lambda}\right) \quad .. \quad .. \quad .. \quad (52)$$

and remembering that

$$\omega t_0=\omega t-\frac{\omega x}{v}=\omega t-\frac{2\pi x}{\Lambda}$$

the resultant displacement modulation can be put into the form

$$S=\frac{e\phi_A}{m}\left\{\frac{\lambda}{2\pi(u-v)}\right\}^2\left[\sin\left(\omega t-\frac{2\pi x}{\Lambda}\right)(1-\cos\theta)-\cos\left(\omega t-\frac{2\pi x}{\Lambda}\right)(\sin\theta-\theta)\right] \quad .. \quad (53)$$

where $\theta=\frac{2\pi x}{\lambda}\left(\frac{u}{v}-1\right)$ as before.

By Equ. (14) this is equivalent to a density modulation

$$I=I_0+\frac{\omega I_0 e\phi_A}{vm}\left\{\frac{\lambda}{2\pi(u-v)}\right\}^2\left[\cos\left(\omega t-\frac{2\pi x}{\Lambda}\right)(1-\cos\theta)+\sin\left(\omega t-\frac{2\pi x}{\Lambda}\right)(\sin\theta-\theta\right] \quad .. \quad (54)$$

The elemental voltage induced in a line element dx_1 at x_1 is found by inserting (54) into (36) and is written

$$dV_1=-\frac{\alpha Z_0\omega^2 I e\phi_A}{2v^2 m}\left\{\frac{\lambda}{2\pi(u-v)}\right\}^2\left[-\sin\left(\omega t-\frac{2\pi x}{\Lambda}\right)(1-\cos\theta)+\cos\left(\omega t-\frac{2\pi x}{\Lambda}\right)\sin\theta-\theta)\right]dx_1 \quad .. \quad .. \quad .. \quad .. \quad .. \quad (55)$$

where θ stands now for $\frac{2\pi x_1}{\lambda}\left(\frac{u}{v}-1\right)$

Taking the phase shifts into account, the voltage at the point x_2 will be

$$dV_{12}=-\frac{\alpha Z_0 I_0\omega^2 e\phi_A}{2v^2 m}\left\{\frac{\lambda}{2\pi(u-v}\right\}^2 \times\left[-(1-\cos\theta)\sin\left(\omega t-\frac{2\pi x_1}{\Lambda}-\frac{2\pi}{\lambda}(x_2-x_1)\right)+(\sin\theta-\theta)\cos\left(\omega t-\frac{2\pi x_1}{\Lambda}-\frac{2\pi}{\lambda}(x_2-x_1)\right)\right]dx_1 \quad .. \quad .. \quad .. \quad .. \quad .. \quad (56)$$

V_2 is the sum of all the voltages generated between $x_1 = 0$ and $x_1 = x_2$, and is obtained by integrating Equ. (56) between these limits, which gives

$$V_2=+V\frac{2\pi^3\alpha^2 Z_0 I_0}{V_0}\frac{x^3}{\lambda\Lambda^2}\left[\frac{2\sin\theta/2}{\theta/2}-2\cos\theta/2\right]\theta^{-2}\sin\left(\omega t-\frac{2\pi x}{\lambda}\left(\frac{u}{v}+1\right)\right) \quad .. \quad .. \quad (57)$$

where $\theta=\frac{2\pi x}{\lambda}\left(\frac{u}{v}-1\right)$ as before.

The function of θ occurring in (57) has been plotted in Fig. 1.

When $u \to v$,

$$V_2=+V\frac{2\pi^3\alpha^2 Z_0 I_0 x^3}{3!\,V_0\lambda^3}\sin\left(\omega t-\frac{2\pi x}{\lambda}\right) \quad (58)$$

It is worth while to recall that the original signal voltage was of the form

$$V\cos\left(\omega t-\frac{2\pi x}{\lambda}\right)$$

Similarly it can be shown that this secondary voltage described by Equ. (58) gives rise to a third-order voltage of the form

$$-V\frac{4\pi^6\alpha^4 Z_0^2 I_0^2 x^6}{6!\,V_0^2\lambda^6}\cos\left(\omega t-\frac{2\pi x}{\lambda}\right) \quad .. \quad (59)$$

and so forth, *ad infinitum*.

It will be observed that each voltage is 90° out of phase with the previous one; and the amplitude of each can be obtained from the previous one by multiplying by some factorials and a dimensionless term.

$$\frac{2\pi^3\alpha^2 Z_0 I_0 x^3}{V_0\lambda^3} \quad .. \quad .. \quad (60)$$

Let this term be denoted by z^3; then summing all the partial voltages and taking account of phases by the symbol j the total resultant voltage V_T can be written in a series :

$$V_T = V\left[1 - (-j)\frac{z^3}{3!} + (-j)^2\frac{z^6}{6!} - (-j)^3\frac{z^9}{9!} + \ldots\ldots\right] \quad .. \quad .. \quad (61)$$

This can be separated into a real and an imaginary part :

$$V_T = V\,[R + jX]$$

The summation of this series has been carried out by Mr. H. Ashcroft, who gives the following expressions :

$$R = \tfrac{1}{6}\sum_{m=0}^{5} e^{ze^{(2m+1)j\pi/6}} = \tfrac{2}{3}\cosh\frac{\sqrt{3}}{2}z\cos\frac{z}{2} + \tfrac{1}{3}\cos z$$

$$X = \frac{d^3R}{dx^3} = +\tfrac{2}{3}\cosh\frac{\sqrt{3}}{2}z\sin\frac{z}{2} - \tfrac{1}{3}\sin Z \quad (62)$$

We are chiefly interested in the (amplitude)^2 of V_T and this can be expressed in closed form by

$$V_T^2 = V^2\left[R^2 + X^2\right] = V^2 \cdot \tfrac{1}{9}\left[\left(2\cosh\frac{\sqrt{3}}{2}z + \cos\tfrac{3}{2}z\right)^2 + \sin^2\tfrac{3}{2}z\right] = V^2\Phi \quad .. \quad .. \quad (63)$$

which tends to $V^2 \cdot \tfrac{1}{9}\,e^{\sqrt{3}z}$ .. (64)

as z is made large.

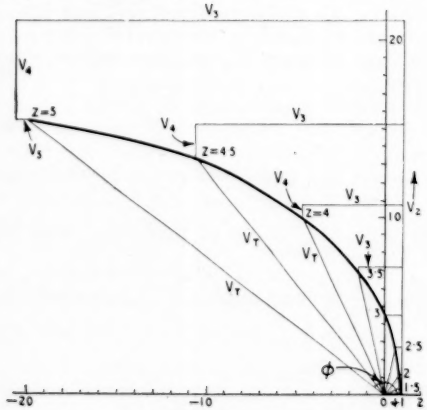


Fig. 5. *Partial and total voltages for various values of z ; u = v. The initial voltage V is represented by the distance o to + 1.*

Φ is simply the power amplification A of the tube as a function of all the parameters of z, and has been plotted in Fig. 6. When z is small, the series can be stopped at the second terms and we have

$$A = 1 + \frac{z^6}{(3!)^2} \quad .. \quad .. \quad .. \quad (65)$$

all the higher-order voltages contributing only to a negligible extent.

However, in practice, large values of z will be more interesting, and we observe the important result that the wave can be then considered to grow exponentially with distance, since z is simply proportional to distance. Thus the electron beam introduces negative attenuation into the line.

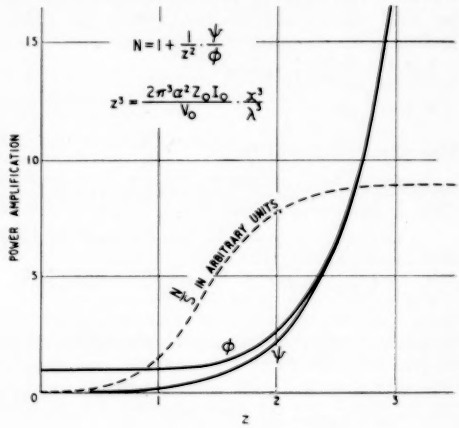


Fig. 6. *Φ and Ψ and noise/signal ratio N/S are shown as functions of z.*

The total and partial voltages in their phase relationship as a function of z have been plotted on the vectorial diagram of Fig. 5. The phase angle of the total voltage is given by

$$\tan\phi = \frac{X}{R} \quad .. \quad .. \quad .. \quad (66)$$

which tends to $+\tan\frac{z}{2}$ for large values of z.

9.—Total Noise Voltage

In the case of axial interaction of beam and fields and when $u = v$ it has been shown [see Equ. (50)] that the first-order shot noise voltage induced in the line is described by

$$\overline{V_2}^2 = \frac{\pi^2\alpha^2 Z_0^2\, 2eI_0\, df\,.\,\Gamma^2 x^2}{\lambda^2} \quad .. \quad (67)$$

Writing

$$\overline{V_2} = \pi\alpha Z_0\sqrt{2eI_0\,df\Gamma^2}\cdot\frac{x}{\lambda}\cos\left(\omega t - \frac{2\pi x}{\lambda}\right) \quad .. \quad .. \quad (68)$$

for the r.m.s. shot-voltage, we obtain the resultant density-modulation of the beam by the use of Equ. (15) ; the voltage induced

by this in the line is found as before by the use of (36), etc., and is

$$\overline{V_2} = -\frac{2\pi^4\alpha^3 Z_0^2 I_0 \sqrt{2eI_0 df}\,\Gamma^2 x^4}{4!\,V_0}\frac{x^4}{\lambda^4}$$
$$\sin\left(wt - \frac{2\pi x}{\lambda}\right) \quad .. \quad .. \quad (69)$$

Similarly we can find the next order shot-voltage and so on *ad infinitum*.

The sum of all individual voltages can be expressed by the series :

$$V_T^* = \frac{|\overline{V_2}|}{z}\left\{z + (-j)\frac{z^4}{4!} + (-j)^2\frac{z^7}{7!} + (-j)^3\frac{z^{10}}{10!} + (-j^4)\frac{z^{13}}{13!} + \ldots\right\} \quad (70)$$

where $z^3 = \dfrac{2\pi^3\alpha^2 Z_0 I_0 x^3}{V_0\lambda^3}$ as before. Equ. (70) can again be separated into a real and an imaginary part,

$$V_T^* = \frac{|\overline{V_2}|}{z}(R^* + jX^*)$$

where

$$R^* = \tfrac{1}{3}\left(\cosh\frac{\sqrt{3}}{2}z\sin\frac{z}{2} + \sqrt{3}\sinh\frac{\sqrt{3}}{2}z\cos\frac{z}{2} + \sin z\right) \quad .. \quad .. \quad ..$$
and
$$X^* = \tfrac{1}{3}\left(\cosh\frac{\sqrt{3}}{2}z\cos\frac{z}{2} - \sqrt{3}\sinh\frac{\sqrt{3}}{2}z\sin\frac{z}{2} + \cos z\right) \quad .. \quad .. \quad .. \quad (71)$$

while
$$(\overline{V_T^*})^2 = \frac{\overline{V_2}^2}{z^2}\tfrac{1}{9}\left\{\left[\cosh\frac{\sqrt{3}}{2}z - \cos\frac{z}{2}\right]^2 + \left[\sqrt{3}\sinh\frac{\sqrt{3}}{2}z - \sin Z\right]^2\right\} \quad .. \quad (72)$$
$$= \frac{\overline{V_2}^2}{z^2}.\Psi.$$

This is the mean-square amplitude of the shot-voltage introduced into the line by the electron beam. Ψ has been plotted in Fig. 6. For small values of z, the series can be stopped at the first term and we have

$$(\overline{V_T^*})^2 = \overline{V_2}^2$$

Hence at first the shot voltage amplitude increases linearly with distance.

Of greater interest is the behaviour of the tube for large values of z. Ψ then approaches

$$\tfrac{1}{9}\,e^{\sqrt{3}z} \quad .. \quad .. \quad .. \quad (73)$$

which is identical with the value of Φ when z is large.

The mean-square shot-voltage can then be written

$$(\overline{V_T^*})^2 = \tfrac{3}{\sqrt{2}}\,e\,\Gamma^2\,df\,.\,I_0^{\frac{1}{3}}\alpha^{\frac{2}{3}}Z_0^{\frac{1}{3}}V_0^{\frac{1}{3}}\tfrac{1}{9}\,e^{\sqrt{3}z} \quad (74)$$

Thus the noise power induced into the line by the electron beam also grows exponentially with distance, when z is large.

10.—Signal-to-Noise Ratio of an Amplifier

The signal-to-noise performance of a tube is commonly defined by a noise factor N, which is given by

$$N = \frac{\text{signal-to-noise ratio at input}}{\text{signal-to-noise ratio at output}}$$

The mean-square noise voltage at the input is $KT\,df\,.\,Z_0$. If we arbitrarily make the signal (voltage)^2 at the input equal to the noise (voltage)^2, that is also $KT\,df\,.\,Z_0$, then the signal (voltage)^2 at the output is $A.KT\,df\,Z_0$. The noise (voltage)^2 at the output, however, is given by the sum of the amplified input noise (voltage)^2 $A.\,KT\,df\,Z_0$ and the shot (voltage)^2 introduced by the electron beam ; i.e., expression (72).

Since the numerator of N is unity, we have

N = Noise-to-signal ratio at output.

$$= \frac{A\,.\,KT\,df\,Z_0 + \dfrac{\overline{V_2}^2}{z^2}.\Psi}{A\,.\,KT\,df\,Z_0} \quad .. \quad (75)$$

A, however, is equal to Φ and thus

$$N = 1 + \frac{\overline{V_2}^2}{Z^2}\cdot\frac{1}{KT\,df\,.\,Z_0}\cdot\frac{\Psi}{\Phi} \quad .. \quad (76)$$

When z is large, Ψ/Φ approaches unity and

$$N = 1 + \frac{2e\Gamma^2}{KT}\,(\tfrac{1}{4}\alpha^2 Z_0 I_0 V_0^2)^{\frac{1}{3}} \quad .. \quad (77)$$

For sufficient amplification z has to be at least 3. In that case, Equ. (77) gives the noise factor of the tube to a good approximation.

11.—Experiments

11.1 *Preliminary Investigations.*

Among various structures which suggested themselves for the purpose of slowing down a wave, the simplest seemed to be a coaxial line with the inner conductor coiled into a helix. Accordingly such a line was built and investigated for its propagation velocity, attenuation and characteristic impedance, using a source of 10 cm wavelength. The propagation velocity was deduced from a measurement of standing waves in the line and it was found that the wave travels along

the turns of the helix with approximately the velocity of light. The attenuation constant α' of the line was deduced from the observed Q-value when short-circuited at both ends, and the characteristic impedance Z_0 was estimated from the standing-wave ratio in a line of known impedance connected to the helical line at one end, the other end being non-reflectively terminated. Typical values of a line having a propagation velocity of one-tenth of that of light, of 18 s.w.g. copper wire and 5 turns/cm are :

$$\alpha' \approx 2 \text{ db/metre}$$
$$Z_0 \approx 500 \text{ ohms}$$

11.2. *Experiments on the Interaction of Wave and Beam*

To be able to arrive at an estimate of the performance of a tube based on the interaction between an electron beam and a travelling wave, we needed to know the actual field strengths, axial and transverse, in the helical line for a given energy in the wave. This knowledge was gained by a series of experiments using the arrangement shown in Fig. 7. Means had to be found for feeding power into, and out of, the helical line without interfering with an electron beam which it was proposed to inject, alternatively, along the axis of the helix, and through the space between helix and the outer conductor. The structure shown was found to function satisfactorily, the rhumbatron-like attachments enabling good matching to be obtained between input and output line on the one hand and the helical line on the other.

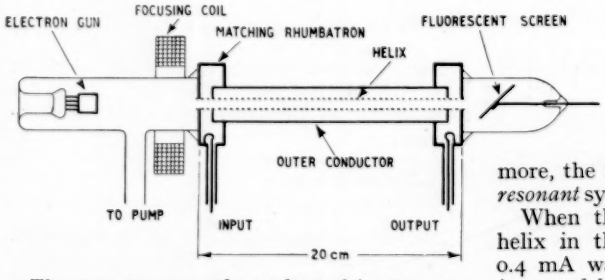


Fig. 7. *An experimental tube for determining the axial and transverse field strengths.*

The structure was then adapted for evacuation, and an electron beam shot through the space between helix and outer conductor. At the far end was placed a fluorescent screen upon which a luminous spot was formed by the electron beam. When r.f. power was sent through the helix and the beam voltage adjusted to the value of 2,400 volts, the luminous spot drew out into a line, roughly in a radial direction. From the magnitude of the deflection, and the amount of r.f. power an average value for the transverse field strength between helix and outer conductor could be deduced using Eqs. (1), (4) and (9). The appropriate weakening factor τ was found to be about 1/4.

Next, a weak beam was shot along the axis of the helix, and the maximum voltage increments, given to the electrons by a known amount of r.f. power, were measured by means of a collector at the far end biased negatively with respect to the cathode. The corresponding weakening factor of the axial field, α, was found to be about 1/6 ; this is the factor by which the field along the axis is weaker than that close to the helix.

If $|\phi|$ is the amplitude of the field at the axis, and x the length of the helix, the maximum voltage increment is given by

$$\Delta V = |\phi| x$$

Comparing this with the corresponding process in a rhumbatron of shunt impedance Z, it can be shown that the 'equivalent shunt impedance' of the helical line can be described by the expression

$$Z \text{ equ.} = \frac{4\pi^2 \alpha^2 x^2}{\lambda^2} Z_0$$

Taking practicable values, such as $\alpha = \frac{1}{6}$, $x = 60$ cm, $\lambda = 1$ cm, $Z_0 = 500$ ohms we have

$$Z \text{ equ.} \approx 2{,}000{,}000 \text{ ohms},$$

a value considerably higher than is commonly obtained with rhumbatrons. Furthermore, the helical line is an essentially *non-resonant* system.

When the r.f. power emerging from the helix in the presence of a beam of about 0.4 mA was measured, it was found to be *increased* by 49% over the power emerging in the absence of the beam. This occurred at a beam voltage of 2,440 volts. At 2,200 volts there was *decrease* of 40%.

Next, the density modulation in the beam was measured as a function of beam voltage. This was accomplished by passing the beam through a rhumbatron after it had traversed the helix. The result is shown in Fig. 8.

A number of observations were then made on the amount of shot-noise introduced into the line by the electron beam. When the beam was shot through the space between helix and outer conductor, the noise power was too small to be observed. When the beam was shot along the axis of the helix, however, it could be observed, though the measurements indicated a certain degree of 'smoothness' of the beam, believed to be due to the mechanism of space-charge smoothing near the cathode. If, as a first approximation, Equ. (50) is taken to describe the way shot-noise power is introduced into the line, the 'randomness' of the beam was reduced by a factor Γ^2 of 0.075.

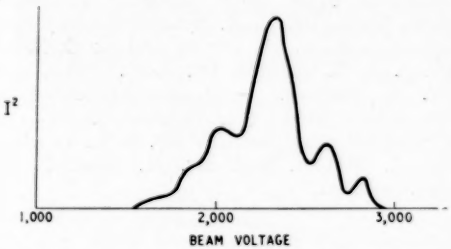


Fig. 8. *Density modulation of the beam current as function of beam voltage.*

11.3. *Experiments on Amplifiers*

Since experiment had shown that more power can be obtained from the helix than is put in, merely by shooting a beam of the right voltage along the axis of the helix, this mode of operation was chosen for the next experiments on the travelling-wave tube as amplifier. In this mode, the length of helical line can be envisaged to be a buncher and catcher combined. The wave in the first portion of the helix velocity-modulates the beam, resulting in a density-modulation or bunching, which in turn induces a wave in the helix of gradually increasing amplitude which supports the initial wave. This is, of course, a rather rough qualitative picture, applying only to small amplification.

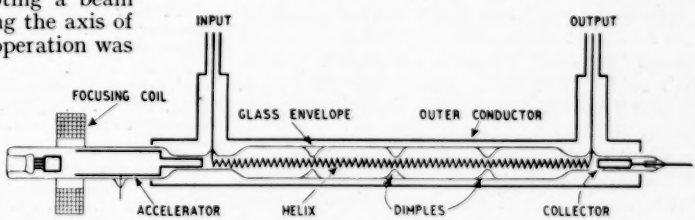


Fig. 9. *The complete travelling-wave tube as an amplifier.*

It was desired to make an amplifier with as good a signal-to-noise performance as possible, and from the theory as developed at that time it appeared that such a tube should be many wavelengths long. Accordingly, measurements were made on a long copper helix enclosed in, and supported by, a glass tube. The attenuation was found to be rather high and this seemed to be due to the presence of the glass near the helix. When a glass envelope a good deal wider than the helix, and supporting the helix only by means of widely spaced dimples was tried, the attenuation was considerably reduced, and such a dimpled tube was then used for the vacuum envelope. The complete tube is shown on Fig. 9. The helix was about 66 cm long and of No. 18 s.w.g. copper wire wound on a $\frac{1}{4}$-in mandrel. The wavelength in the tube was 7.7 mm, corresponding to a beam voltage of 1,830 volts. (The free-space wavelength was 9.1 cm). The cold insertion loss of the tube, due to loss in the copper and glass, and to mismatches, was about 3.5 db. The electron beam was produced by a standard cathode-ray tube electron gun and focused magnetically by means of a short coil outside the tube. A thick soft-iron tube surrounded the main portion of the tube, to prevent stray magnetic fields from deflecting the long and thin electron beam.

With this tube, a net power gain of 6 was obtained with a beam current of 119 μA and the overall noise factor of the receiver of 16 db was improved by 2 db.

There was evidence of oscillations being present most of the time at wavelengths not far from the signal wavelength. These oscillations persisted down to beam currents as small as 20 μA.

It was further noticed that the noise factor of the tube depended on the fraction of beam current coming right through to the collector; only when more than 90% of the beam current was collected was there any improvement in the overall signal-to-noise ratio. This is believed to be additional

evidence of the existence of space-charge smoothing in the electron beam.

Next, a helix of different proportions, held in a dimpled quartz tube, was tried using somewhat different input and output matching arrangements. This helix was wound of No. 22 s.w.g. copper wire on a 3/16-in mandrel with about 7 turns per cm and its length was 60 cm. A net power gain of 14 was obtained with a beam current of 40 μA

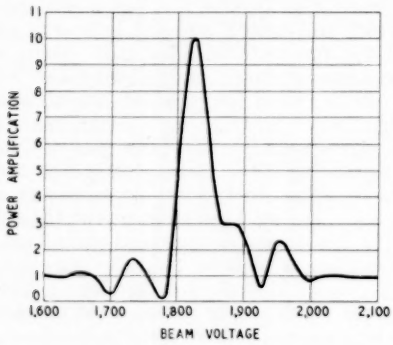


Fig. 10. *Variation of power amplification with beam voltage.*

coming through to the collector out of 50 μA total beam current. The overall noise factor of tube plus receiver was then improved by 4 db which corresponds to a noise factor of the tube by itself of 11 db.

Since the operation of the tube depends on the fact that wave and electrons travel with about the same velocity, the beam voltage has to be controlled within narrow limits. A graph showing power amplification against beam voltage is shown in Fig. 10.

These experiments fully confirmed the expectation that an amplifier could be built, on the principle of the interaction between travelling waves and an electron beam and that such an amplifier could be made with a signal-to-noise performance sufficiently promising to warrant further interest and study.

12.—Acknowledgments

The work forming the subject of this paper has been carried out for the Admiralty and is published by permission of the Board of Admiralty. It was initiated at the Physics Department, Birmingham University in 1942 and carried on, after a move in 1944, at the Clarendon Laboratory, Oxford University. All the investigations described in this paper were completed before the end of 1944.

The writer is indebted to many for very helpful discussions and interest; at Birmingham these were chiefly: Professor P. B. Moon, Dr. R. R. Nimmo, Professor J. Sayers and Dr. G. Voglis, and at Oxford: Dr. J. H. E. Griffiths, Dr. A. H. Cooke, and Dr. B. Bleaney. Very able assistance in the actual work, experimental as well as theoretical, was given by Mr. E. E. Vickers, Mr. J. Hatton and Mr. H. Ashcroft at various times.

RADIOLYMPIA

The fifteenth National Radio Exhibition opens at Olympia on October 1st, but there is to be a private view on September 30th. It will be open daily, except Sundays, from October 1st to 11th, between 11 a.m. and 10 p.m. Admission will be 2s. 6d.

The number of exhibitors is 186 and, in addition to broadcasting and television equipment, navigational aids and industrial electronic apparatus are now included.

The non-commercial exhibitors include the Ministry of Supply and the Metropolitan Police Force.

Wireless Engineer stand number is 242.

Institution of Electrical Engineers

The new president of the I.E.E. is P. Good, C.B.E., director of the British Standards Institution. He takes office on September 30th. Among the new members of the council are: T. E. Goldup, a director of Electronic Transmission Equipment, Ltd., and H. L. Kirke, C.B.E., head of the B.B.C. Research Department.

In the Radio Section the new chairman is C. E. Strong, O.B.E., B.A.I. (Standard Tele communication Laboratories) and the vice-chairman is F. Smith, O.B.E., (M.O. Valve Co.). The ordinary members are: C. F. Booth (G.P.O. Research Station), H. W. Forshaw, O.B.E. (M. of S., Directorate of Telecommunications Research and Development [Defence]), E. L. E. Pawley, M.Sc. (Eng.)., (B.B.C.), and J. A. Ratcliffe, O.B.E., M.A. (Cavendish Laboratory).

BELLING & LEE

We are asked to correct an error which occurred in the Belling-Lee advertisement in the August issue. The first paragraph implied that the spigotless version of the B8A valve was known as the B8B. This is incorrect. The spigotless version is an accepted variation and comes within the standard specification of the B8A.

PERMEABILITY OF DUST CORES

Horatio W. Lamson

(*General Radio Company, Cambridge, Massachusetts*)

IN the February 1947 issue of *Wireless Engineer*, P. R. Bardell cites evidence indicating that the ferro-magnetic granules in dust cores have dimensions along the flux exceeding their transverse dimensions, thus affording a larger composite or effective permeability μ_c for a given volumetric concentration ratio p than could be attained with symmetrical granules such as hypothetical cubes or spheres.

The writer was interested in investigating Bardell's hypothesis somewhat further by considering, purely for convenience, a centimetre cube of the composite core material traversed by flux perpendicular to two opposite faces. The reluctance of this flux path would then be $1/\mu_c$.

For purposes of analysis, consider that the individual granules have the form of identical right cylinders which are all perfectly aligned, end to end, along the flux with appropriate axial and transverse separations. A portion of the flux will thus traverse the maximum number of these granules from face to face, while the remainder of the flux will follow a wholly non-magnetic path having a permeability of unity. Fringing effects are to be considered negligible. On this hypothesis it will be legitimate to consider the granules congregated, by an axial and a transverse displacement, into a solid ferro-magnetic cylinder having an axial length λ, a uniform cross-section A, and a volume $p = \lambda A$. The non-magnetic flux path will then have a length of unity and a cross-section $1 - A$. If μ is the normal permeability of the ferro-magnetic material at the induction which it carries, the total reluctance to the flux traversing the granules will be :

$$\frac{\lambda}{\mu A} + \frac{1-\lambda}{A} = \frac{\mu - \lambda(\mu - 1)}{\mu A}$$

and the reluctance of the non-magnetic path will be $1/(1-A)$.

Equating the parallel combination of these two reluctances to $1/\mu_c$ and noting that the concentration ratio p equals $\lambda A/1$, it follows that the composite permeability is given by :

$$\mu_c = 1 + \frac{p}{k - \lambda} \quad .. \quad .. \quad .. \quad (1)$$

wherein k is the ratio of the normal to the intrinsic permeability $(\mu - 1)$ of the ferro-magnetic material :

$$k = \frac{\mu}{\mu_i} = \frac{\mu}{\mu - 1} \quad .. \quad .. \quad .. \quad (2)$$

It should be noted that specific values of μ, μ_c, and p demand a unique value of λ given by :

$$\lambda = k - \frac{p}{\mu_c - 1} \quad .. \quad .. \quad .. \quad (3)$$

and a total ferromagnetic cross-section p/λ, regardless of the configuration of the cross-section of the individual granules and of their distribution in the cross-section of the composite core.

In the design of toroidal dust-core inductors, the fractional part of μ achievable as a composite permeability is significant. From Equation (1) :

$$\frac{\mu_c}{\mu} = \frac{k - \lambda + p}{\mu(k - \lambda)} \approx \frac{1 + \dfrac{p}{1 - \lambda}}{\mu} \quad .. \quad (4)$$

the third member assuming a sufficiently large μ so that k becomes unity. It is of interest to note that, to obtain a desired value of μ_c/μ with a specific concentration, the value of λ must be increased when a ferro-magnetic material having a larger permeability is chosen.

The ratio, λ, of the flux path in the granules to the total flux path must lie within the limits :

$$p \leqslant \lambda \leqslant 1 \quad .. \quad .. \quad .. \quad (5)$$

Taking λ at its maximum value, Equation (1) becomes :

$$\text{Max } \mu_c = 1 + p(\mu - 1) \quad .. \quad .. \quad (6)$$

which defines the " concentric rings " curve given by Bardell ; while taking λ equal to its minimum value gives :

$$\text{Min } \mu_c = 1 + \frac{p}{k - p} = \frac{\mu}{p + \mu(1 - p)} \quad .. \quad (7)$$

which defines his " solid ring with airgap " curve.

The second member of (7) was derived by the writer (*Wireless Engineer*, November 1946) to give the effective permeability assuming uniform cubical granules having

a random distribution but with opposite faces perpendicuar to the flux.

In Figs. 1–3 the Bardell curves of μ_c versus p, Equations (6) and (7), are given—assuming normal permeabilities of 100, 1000, and 10,000. Between these limiting values, a series of graphs for specific values of λ have been computed from Equation (1). These figures demonstrate the relationships between the parameters μ_c, p, and λ for widely differing values of μ.

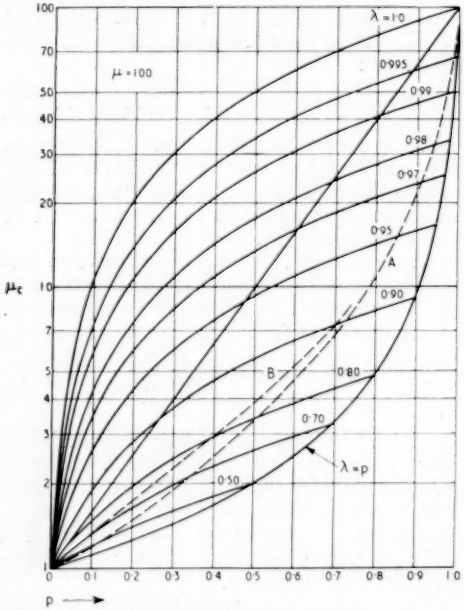


Fig. 1. *Bardell curves of μ_c versus p for $\mu = 100$. Curves A and B represent Equations* (10) *and* (11) *respectively.*

The linear diagonal corresponds to the exponential relation :

$$\mu_c = \mu^p \qquad .. \quad .. \quad .. \quad (8)$$

It can be shown that the λ curve which is tangent to (8) at $p = 0$ is given by :

$$\lambda = k - 1/(\log_e \mu) \qquad .. \quad .. \quad .. \quad (9)$$

These have the values $\lambda = 0.793$, 0.856, 0.892, for $\mu = 100$, 1000, 10,000, respectively. Any larger value of λ can satisfy (8) for a specific p value ; smaller values of λ cannot satisfy (8).

The dashed curves labelled A represent the equation :

$$\mu_c = 1 + p/(k - \sqrt[3]{p}) \qquad .. \quad .. \quad (10)$$

which was previously derived by the writer, (*Wireless Engineer*, November 1946), assuming uniform cubical granules perfectly aligned both axially and transversely.

In *Wireless Engineer*, June 1946, R.E. Burgess gives the equation :

$$\mu_c = 1 + 3p\Big/\left(\frac{\mu + 2}{\mu - 1} - p\right) \qquad .. \quad (11)$$

which is valid for spherical granules at lower concentrations. The dashed curves labelled B depict Equation (11) extended up to the maximum concentration $p = 0.74$ attainable with uniform spheres.

It would seem that the curves A and B together stipulate the *maximum* values of μ_c attainable assuming a uniform distribution of identical symmetrical granules, such as spheres or cubes or the statistical equivalent thereof. Bardell shows a curve for cubic granules with truncated corners which lies slightly above A and is an approximate extrapolation of B. Therefore, for a given concentration, any larger empirical value of μ_c obtained would indicate asymmetrical granules having their major dimension more parallel with than transverse to the direction of the flux.

Consequently, whenever λ exceeds $\sqrt[3]{p}$, i.e., from (3), if the ratio :

$$\frac{\lambda}{\sqrt[3]{p}} = \frac{k}{\sqrt[3]{p}} - \frac{\sqrt[3]{p^2}}{\mu_c - 1} \qquad .. \quad .. \quad (12)$$

exceeds unity, the cubical granules must have undergone a longitudinal expansion resulting in a longitudinal composite permeability exceeding that given by Equation (10).

Bardell considers the granules to have the form of rectangular slabs with dimensional ratios 10 : 2 : 1 and gives the curve C in Fig. 2 for the composite permeability. It is convenient to define m and n as the ratios of the longitudinal dimension of the granular slabs to their major and minor transverse dimensions, respectively, $(m < n)$. The Bardell slabs, therefore, have the elongation ratios $m = 5$ and $n = 10$. To compute the composite permeability when m differs from n, it is not only necessary to know the values of these ratios but it is also essential to specify some hypothesis defining the distribution of the granules throughout the transverse cross-section of the core.

Not knowing what hypothesis Bardell used in computing curve C, the writer made an assumption which is logical from various points of view—namely, that the insulation between adjacent granules has a uniform thickness at all points. It can then be

shown that the extent of the ferro-magnetic paths in the major and minor transverse directions are :

$$\text{Major } \lambda' = \frac{\lambda}{\lambda + m(1-\lambda)} \quad .. \quad (13)$$

$$\text{Minor } \lambda'' = \frac{\lambda}{\lambda + n(1-\lambda)} \quad .. \quad (14)$$

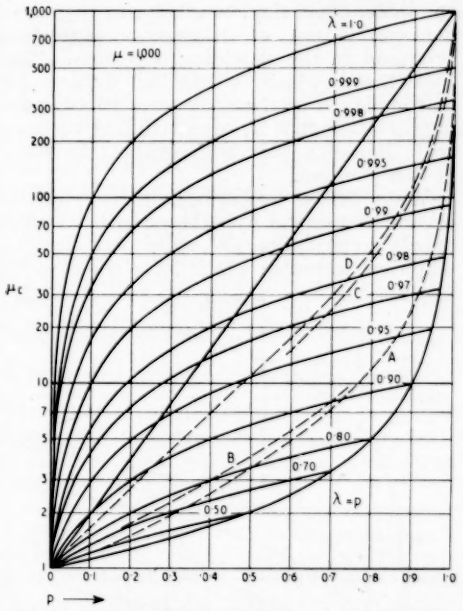


Fig. 2. *The Bardell curves for* $\mu = 1{,}000$. *Curves A and B represent Equations* (10) *and* (11) *while curve C is for rectangular slabs of ratios* 10 : 2 : 1. *Curve D gives values of longitudinal* μ_c.

For analysis these slabs may then be congregated into a rectangular prism having the volume :

$$p = \lambda\lambda'\lambda'' \quad .. \quad .. \quad .. \quad (15)$$

Substituting (13) and (14) into (15) gives the cubic equation involving the longitudinal λ only.

$$\lambda^3 - p\,[\lambda^2(1 - m - n + mn) + \lambda(m + n - 2mn) + mn] = 0 \quad (16)$$

It may be noted that this relationship between λ, p, and the elongation ratios is independent of the permeability of the ferro-magnetic material. Introducing Bardell's m and n values into (16) :

$$\lambda^3 - 36p\lambda^2 + 85p\lambda - 50p = 0 \quad .. \quad (16a)$$

Applying Cardan's solution to (16a), the values of λ were obtained for specific values of p. For p less than about 0.22, Equation (16a) has only one real root ; for larger values of p it has three real roots, only one of which is applicable to this problem. These data were then substituted in (1) to obtain the values of longitudinal μ_c, depicted by curve D in Fig. 2. It is apparent that Bardell must have used a somewhat different distribution hypothesis in obtaining curve C.

The assumption of elongated-slab granules for which m and n exceed unity leads to transverse values λ' and λ'' which differ (generally being smaller) from the longitudinal λ. Consequently, when λ' or λ'' replaces λ in Equation (1), smaller values for the composite permeabilities μ_c' and μ_c'' for flux in the major and minor transverse directions through the medium are to be expected. Again considering the Bardell slabs and assuming uniform insulation thickness, the following ratios of the longitudinal to the two transverse permeabilities were computed when $m = 5$ and $n = 10$.

TABLE 1.

p	..	..	..	μ_c/μ_c'	μ_c/μ_c''
0.9	..	..	..	4.247	7.848
0.8	..	..	..	4.206	7.315
0.7	..	..	..	3.952	6.401
0.6	..	..	..	3.619	5.447
0.5	..	..	..	3.217	4.345
0.4	..	..	..	2.745	3.523
0.3	..	..	..	2.238	2.651

A measurement of these three composite permeabilities should furnish a confirmation of this hypothesis of granular elongation. To investigate this question, the writer cut a centimetre cube from a molybdenum-permalloy toroidal dust core manufactured by the Western Electric Company and measured the initial values of its composite permeability in three directions, using a technique which permitted a reasonably accurate evaluation of permeability ratios and allowed for any leakage flux. The core had a known longitudinal μ_c of 125 which, according to Legg and Given, satisfies (8) *empirically* when μ is taken as 220 therein, (*Bell System Technical Journal*, July 1940). Consequently, the material tested had a volumetric concentration of 0.895. Using this value of p, the data given in Table II were computed for each of four assumed values of the initial permeability of the molybdenum-permalloy. The transverse permeability μ_c' (corresponding to m) was measured along the axis of the toroid, while μ_c'' (corresponding to n) was measured along the radius of the toroid.

These data confirm Bardell's hypothesis, by indicating a definite granular elongation, although with distinctly smaller m and n values than he proposes.

TABLE II.

Assumed μ	1,000	2 000	5,000	10,000
μ_e	125	125	125	125
μ_e'	86.7	96.0	103.5	106.5
μ_e''	39.8	50.4	62.1	67.1
μ_e/μ_e'	1.44	1.30	1.21	1.17
μ_e/μ_e''	3.14	2.48	2.01	1.86
λ	0.99378	0.99328	0.99298	0.99288
λ'	0.99056	0.99108	0.99147	0.99162
λ''	0.97793	0.98238	0.98555	0.98656
m	1.523	1.330	1.217	1.178
n	3.607	2.652	2.074	1.900

Bardell depicts this granular elongation to be produced by an actual crushing of initially symmetrical granules. If a hydrostatic pressure exists during the compression of the toroidal cores, it is interesting to speculate whether his slabs might not be formed by the direct cohesion of sufficient numbers of symmetrical and undistorted granules to give statistically averaged values of m and n exceeding unity.

It should be noted that smaller values of transverse μ_e are not, *a priori*, proof of slab granules. The writer wishes to point out that, to explain the μ_e values encountered in practice which lie above curve A or curve B extrapolated (truncated cubes), it is not imperative to assume slab granules with elongation ratios exceeding unity. As an alternative hypothesis, it is conceivable that the process of core formation results in a larger average thickness of insulation between uniform cubical granules in a transverse than in a longitudinal direction. Equation (10) assumes uniform insulation in all directions between aligned cubes so that $\lambda = \lambda' = \lambda'' = \sqrt[3]{p}$. Had the insulation thickness in the major and minor transverse directions been, respectively, s and t times that in the longitudinal direction ($s<t$), then $\lambda>\lambda'>\lambda''$ under the condition of (15), and it can be shown that :

$$\lambda' = \lambda/\{\lambda + s(1-\lambda)\} \quad .. \quad .. \quad (17)$$

$$\lambda'' = \lambda/\{\lambda + t(1-\lambda)\} \quad .. \quad .. \quad (18)$$

Comparing (17) and (18) with (13) and (14) and assuming $s = 5$ and $t = 10$, it is apparent that the longitudinal μ_e values given by curve D and the permeability ratios tabulated above would be obtained.

Both of the foregoing hypotheses are based on the assumption of uniform granules prismatic in shape. On the other hand, if one departs from the consideration of uniform granules, existing values of μ_e could conceivably be obtained by a close packing of spherical granules having some statistical distribution of diameters which would permit a maximum concentration exceeding 0.74 and approaching unity. The smaller values of transverse μ_e observed by the writer could then be attributed to greater average values of transverse separation produced in the moulding operation. This hypothesis, which would be difficult of mathematical analysis, appears to be quite logical although it is, admittedly, at variance with Bardell's photomicrographic evidence of slab granules.

While debate is still open as to which of several theoretical hypotheses best describes actual performance, the writer will cite two empirical formulae which have come to his

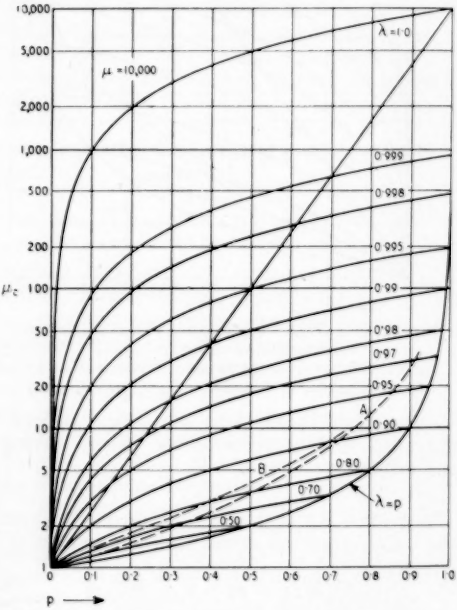


Fig. 3. *Bardell curves for $\mu = 10{,}000$ with equations (10) and (11) represented by curves A and B.*

attention. The first of these, applicable to molybdenum permalloy, is due to Legg and Given and, as mentioned above, consists of writing 220 for μ in (8). The second is due to Leigh Page and was cited by Burgess in *Wireless Engineer*, November 1946. It is obtained from (11) by inserting the empirical factor $(1 - 0.96\sqrt[3]{p})$ in the denominator.

ELECTRO-ENCEPHALOGRAPH AMPLIFIER

By Denis L. Johnston, B.Sc.(Eng.)

(*Continued from page 242, August issue*)

2.2.2. *Low-Frequency Response*

THE coupled-cathode circuit has a property of time constant lengthening that does not appear to be generally recognized. When there is a single self-bias cathode-load resistor R_1, as in Fig. 20(*a*), the input impedance seen from the preceding stage is obviously $2R_3$. However, when the cathode loads are divided, as is necessary for the insertion of a differential control, Fig. 20(*c*), the input impedance is that of each valve functioning individually as a cathode-follower, the individual values being

$$R_1 = \frac{(R_2+R_3+R_5)\,[R_4+r_a+R_2+(R_1+R_2)\,\mu]}{R_4+r_a+R_2+\mu R_1}$$

The value of the differential cathode control alters this impedance very little for it is shunted by $2(R_1+R_2)$ which is itself small compared with the input impedance.

In the circuit used the calculated value of dynamic input impedance is 24 megohms. A slightly lower impedance was observed, probably due to traces of grid current. This can be reduced by changing the circuit constants so that the valves work at a more negative grid potential, at the expense of gain. A stable value of 10 megohms was obtained by placing a stabilizing resistor R as in Fig. 20(*c*): it is ineffective in the position Fig. 20(*d*).

A time constant of 10 seconds is obtained with this dynamic impedance and the push-pull coupling capacitors of 2 μF each (effectively in series). This corresponds to a loss of 3 db at a frequency of one cycle in 60 seconds. In the present design the time constant per stage is reduced to two seconds by choosing a lower value of R when the higher value of time constant is not required.

No special provision is made in the amplifier for the correction of phase shift at very low frequencies, as the long time constant is available if required, but a useful compensating circuit has been noted in the literature[47].

The 2-μF coupling capacitors must be of high quality with an insulation resistance not less than 10^9 ohms. For a steady potential-difference of 100 volts there is then sufficient leakage current to produce 0.1 volt bias across a one-megohm grid leak, an amount

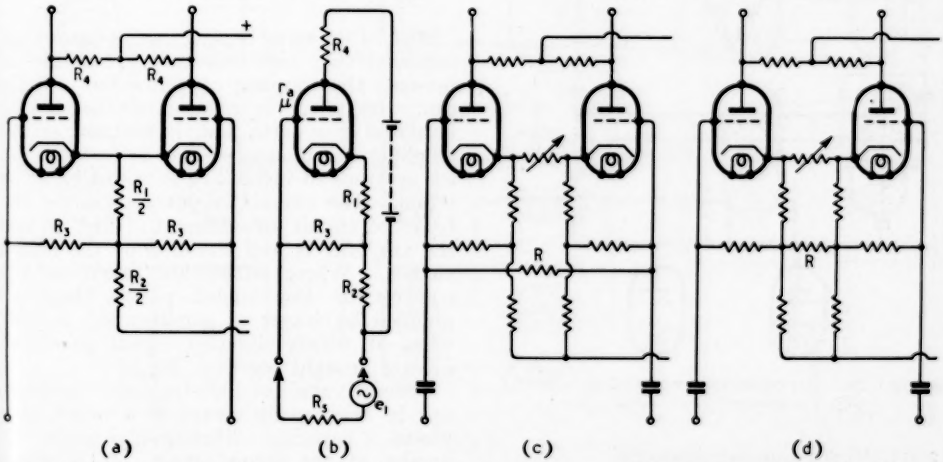


Fig. 20. *Simple coupled-cathode amplifier with self-bias* (a) *and equivalent cathode-follower circuit of one side of amplifier with inphase signals* (b). *A time-constant lengthening arrangement with stabilizing resistor R is shown at* (c) *and an ineffective position for R at* (d).

that is rather high and only tolerable if the capacitor leakage resistance is very stable. Oil-filled paper capacitors of 1,000-volt rating have been found satisfactory.

Time constants of less than the maximum overall value of one second are selected by switching smaller capacitors in series with the 2-μF components at one position of the inter-stage coupling, Fig. 21. To avoid a switching surge when the low-frequency control is operated the pairs of capacitors are connected together differentially by resistors R sufficiently high in value not materially to reduce the overall time constant. The resistor maintains the pairs of capacitors at equal potentials. It is not satisfactory to place leak resistors across individual capacitors rather than differentially, as small inequalities between the two sides of the amplifier will lead to considerable distortion of waveform for short time constants. At all times the 2-μF capacitors remain in series to block the direct potential from the preceding stage, so the switched capacitors need not be of especially high insulation resistance. The overall time constant of 1 second corresponds to a low-frequency response 3 db down at 0.15 c/s.

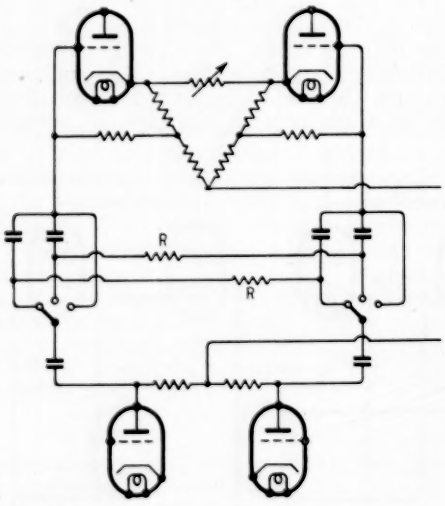


Fig. 21. *Arrangement for selecting value of amplifier time constant.*

2.2.3. *High-Frequency Response.*

The high-frequency response of the circuit shown in Fig. 36 is 3 db down at 10 kc/s and is sufficiently high for viewing biological potentials on an oscilloscope connected to the anodes of the penultimate stage. To obtain a higher frequency response it is necessary to reduce the anode loads, and to increase the current load on the h.t. stabilizer circuits in order to maintain the same amplification.

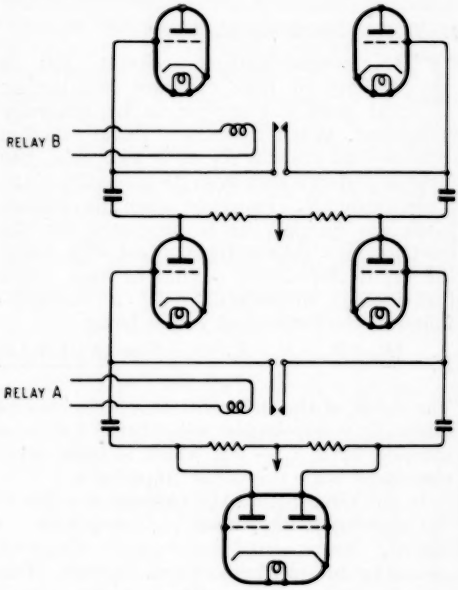


Fig. 22. *Relays A and B for "muting" the capacitor-coupled stages to reduce blocking time.*

Most of the stray capacitance to earth that restricts the high-frequency response is between the coupling capacitor foil and its can. Its effect is reduced if the can is insulated from earth, but the stability of the amplifier is impaired. This is undesirable, for no trace of instability is normally to be found. The chassis layout reproduces the balanced circuit arrangement of Fig. 36 and the amplifier is well screened by the closed chassis. Where cables are taken to the controls on the hinged panels they are grouped by stages of amplification so that wires at widely different signal potentials are not brought together (Fig.2.)

Several values of high-frequency response can be selected by means of a switch that places a capacitor differentially across the anodes at the second stage. It is worth noting that the effective output impedance of this stage is not approximately the value

of the anode loads, but is considerably lower due to the feedback arrangement over the first two stages.

2.2.4. *Reduction of Blocking Time*

A differential amplifier with couplings of long time constant may "block" for a period of a minute or more under certain conditions unless precautions are taken to prevent this nuisance.

Blocking is initiated by an excessive input signal or accidental disconnection of the input, or by the action of operating the gain and frequency response controls. A large signal then passes through the amplifier, driving the valve at one side of each stage into a condition of grid current, and some time is then required before the grid leaks bring the coupling-capacitor potentials back within the working range of the valve grids. Capacitor-coupled stages will "unblock" progressively, and the least blocking time will be realised in an amplifier with few stages capacitor coupled.

For the purposes of electro-encephalography the inputs of a number of amplifiers are connected to electrodes placed on the patient's scalp through a number of contact selector switches shown in Fig. 4. During an examination adjustments are required from time to time and it is while doing this that lengthy "blocking" periods can be very tiresome. It is a general rule that all variable controls should be as far "forward" in the amplifier as possible so that switching surges are subject to the least possible amplification. In the experimental design it was undesirable to place a gain control across the anodes of the fourth stage, because the impedance conditions at this point were not constant, owing to appreciable grid current loading by the grids of the output stage of power amplification.

In this experimental equipment a Recorder ON/OFF switch is provided. Before making an adjustment this is turned to OFF, so disconnecting the outputs of the amplifiers from the recorder. At the same time the switch operation causes the pairs of grids at the fourth and third stages (in that order) to be short circuited by relays fitted in the amplifier chassis. The blocking impulse is thus "caught" and dispersed rapidly, because the time constant at each stage with the relay closed is that given by the anode load and coupling capacitor only. (Fig. 22.)

When the adjustment of the input connections has been completed the recorder switch is turned to ON, and the relays open again in the reverse order, first, the 3rd

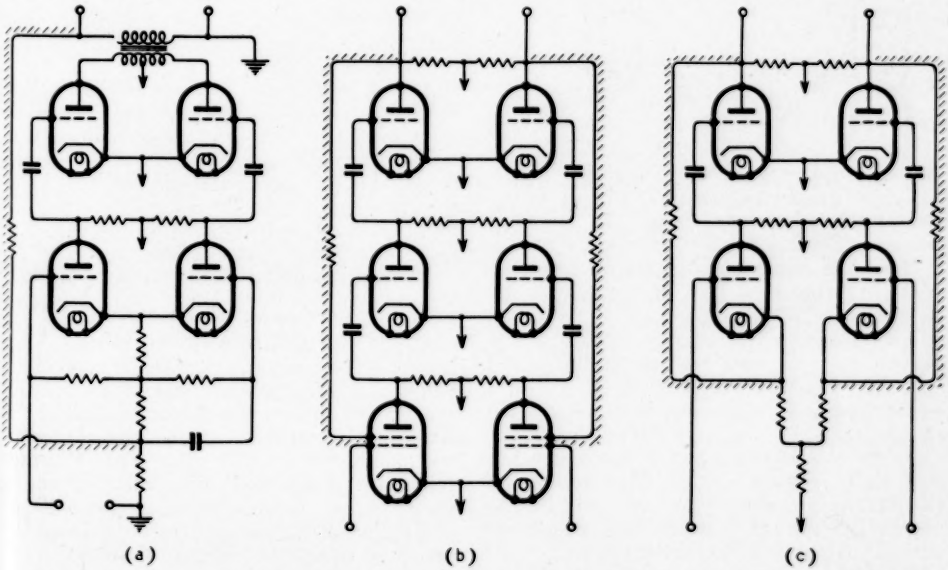


Fig. 23. (a) *Moir's negative feedback circuit;* (b) *negative feedback to grids of first stage;* (c) *negative feedback to first stage cathodes.* (*Feedback paths are shaded.*)

stage and then the 4th stage grids. This timing of the relays is obtained very simply by making the total switch motion 90 degrees and spacing the contacts controlling the relays within this angular motion. No blocking is noticeable if the switch is turned at such a rate that the 90 degree motion takes four seconds.

The grid current is restricted under conditions of blocking by resistors of 47 kΩ in series with the grid connections. Voltage drop across these resistors provides a convenient means of detecting the presence of a faulty valve with appreciable grid current. The degree of blocking is reduced considerably by attention to the static balancing of the direct voltage at differential points in the anode and cathode circuits.

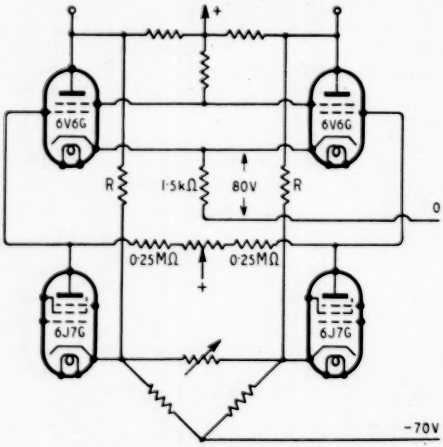


Fig. 24. *Direct-coupled driving (4th) and output stages, with negative feedback by resistors R.*

2.2.5. *Feedback in Differential Amplifiers*

There are two main reasons for applying negative feedback to amplifiers; the improvement of stability, and the reduction of overall distortion. The considerable degeneration of in-phase voltages in differential amplifiers leads to a high degree of stability, but it is not easy to arrange for the reduction of distortion by negative feedback. However, the overall distortion is very much less than in a single-sided amplifier because of the push-pull mode of operation, and for the present purposes the performance is adequate without the provision of feedback.

Moir[48] has shown a method of applying negative feedback to an audio-frequency amplifier where the input is single-sided and this is indicated in Fig. 23(*a*). When the input is differential, feedback must be taken over each side of the amplifier to one of the electrodes of the valves in the first stage, as in Fig. 23(*b*) or (*c*), but it becomes necessary to balance the two feedback paths and the circuit loses the attraction of the degeneratively balanced amplifier.

2.3. *Output Stage*

2.3.1. *Direct-Coupled Drive Circuit*

The output stage is directly coupled to the preceding stage as shown in Fig. 24. The grids of the 6V6G valves are brought up to the same working potential as the anodes of the stage before by a relatively large common cathode load. This arrangement satisfies the requirement mentioned in Section 1.3 that there should be the least possible number of inter-stage capacitor couplings. It would not be easy to obtain the required time constant if this coupling were not direct, because the maximum satisfactory value of grid leak for the 6V6G valve is about 0.25 MΩ.

The anode load at the fourth stage is high for the reasons already discussed and is itself about 0.25 MΩ. A coupling capacitor of 8 μF would then be needed to realize a time constant of two seconds per stage, and the low value of grid leak would entail a 6-db loss of amplification. With direct coupling each 6V6G grid works from a 0.25-MΩ load impedance, and it was found that the static balance conditions remained constant over long periods if the precaution was taken of ageing all new 6J7G and 6V6G valves before use. Change in static balance when the differential cathode control is at maximum attenuation is reduced by the negative feedback circuit through the resistors *R*. Feedback is zero at minimum attenuation.

2.3.2. *Recorder Drive Arrangements*

A balanced-armature moving-iron recorder is normally employed with the present amplifier. The coils are stationary and can accordingly be wound for an impedance of the same order as the anode-load resistance of the pentode-output stage. The recorder coils are connected differentially across the pair of anodes because the wattage dissipation in a multi-pen recorder would be excessive if the h.t. current were fed to the centre point of the coils.

Some forms of recorder incorporate a

double-resonant system to obtain higher frequency response and more constant characteristic of phase shift with frequency than is realized in a single-resonant system. Cut-off occurs at about 100 c/s, and between zero frequency and 100 c/s the response falls off by about 3 db [Fig. 25]. This falling characteristic is desirable for some purposes, but where necessary it can be compensated at the output stage by modifying the cathode circuit as in Fig. 26. The corrected response is given in Fig. 25.

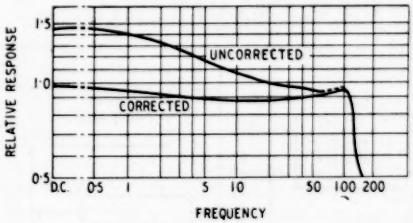


Fig. 25. *Frequency response of recorder and driving amplifier, with and without correcting circuit.*

Rochelle-salt crystal recorders are sometimes employed. They may be driven directly from the anode loads of the output stage. Moving-coil recorders are, by their nature, difficult to design with a coil impedance high enough to match the driving impedance of the anode output from a medium-sized power valve : some recent designs are better in this respect through the use of the higher flux densities obtained when using the newer permanent-magnet materials. Normally the

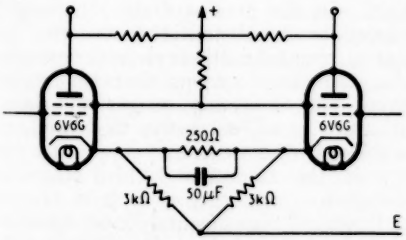


Fig. 26. *Frequency-response correcting circuit applied to cathode of output stage.*

critical damping of the moving-coil system is provided electrically and the driving impedance required in typical commercial designs is of the order of a few hundred ohms. This can be obtained with a cathode-output amplifier. Transformer coupling is impracticable at frequencies of the order of 1 c/s.

The present amplifier can be modified for cathode-output working as in Fig. 27. The impedance of the differential cathode output is approximately 600 Ω, and the voltage swing available without distortion approximately 50 V p-p. The voltage amplification of the output stage is lost when using the cathode-output arrangement, but this is partly compensated by the greater efficiency of a moving-coil recorder.

3. Electronicaly Stabilized Supplies

3.1. *Discussion of Performance*

Degenerative voltage and current stabilizing circuits have recently received a good deal of attention in the literature. Outstanding applications of the technique have been made by Hansen[49], in controlling a 40-kV electrostatic generator constant to 0.02 per cent, and by Vance[50] with an h.t. supply for an electron microscope variable in potential between 20 and 100 kV and stable to 0.007 per cent over an interval of 30 seconds. Hunt and Hickman have stated[51] that the stability obtained in a mass spectroscope with degenerative stabilization was better than with batteries. Brown[52] has described stabilized current and voltage supplies for this instrument.

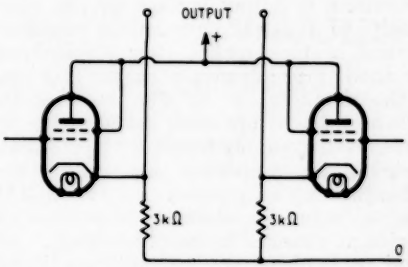


Fig. 27. *Modification as a cathode output amplifier.*

A number of designs of amplifiers that employ stabilized power supplies are noted in Section 1.1 and references[39,53–55]. Hitherto a performance adequate for electro-encephalographic work has not been achieved without batteries for some current or reference voltage supplies. The measures adopted in the present design to eliminate all batteries are outlined below.

The actual performance at the several sections of the high-tension and heater-supply stabilizer circuits is represented in

Fig. 28. The fluctuations that appear at the amplifier output when the mains supply changes abruptly by 1 volt r.m.s. in 230 V is equivalent to 3 μV peak-to-peak signal input, when the amplifier time constant is one second. The fluctuation is less for slow changes in supply voltage, and is, of course, reduced when the amplifier is operated at a time constant less than one second. When advantage is taken of the time-constant lengthening property of the amplifier circuit, to obtain in overall time constant of 5 or 10 seconds, it is advisable to run the equipment from a stabilizing mains transformer capable of reducing the supply-voltage variations by about ten to one.

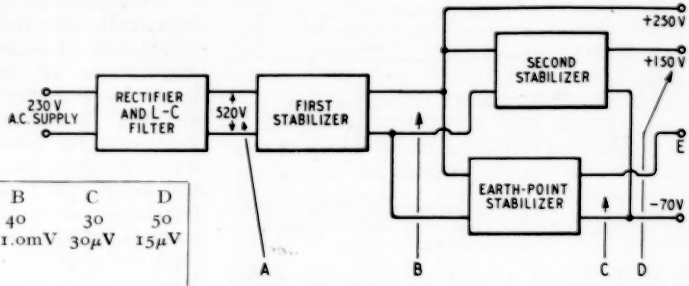


Fig. 28. *Performance at the several stages of h.t. stabilization.*

	A	B	C	D
Output impedance	850	40	30	50
A.C. ripple voltage	0.15V	1.0mV	30μV	15μV
Peak d.c. fluctuation due to 1V r.m.s. abrupt change in 230V a.c. supply	... 2.2V	50mV	500μV	100μV

At an amplifier time constant of one second fluctuations in output voltage are due about equally to residual changes in output of h.t. and heater supplies after stabilization. The anode load adjustment at the first stage of the amplifier, R in Fig. 10, has two slightly different optimum settings, the one where residual supply frequency ripple is at a minimum (an equivalent input signal level of less than 0.3 μV p-p is obtainable), and the other optimum for minimum fluctuation due to abrupt changes in supply voltage. The latter point of balance is found by adjusting R for the minimum change in recorder deflection when the stabilized h.t. voltage is altered slightly by applying an additional load resistor by means of a test jack (" BAL " of Fig. 36.)

3.2. *Requirements of H.T. Stabilizing Circuit*

D.C. stabilizing circuits are of two distinct types, bridge-balanced and degenerative : in addition there are combined arrangements of degenerative circuits with subsidiary balancing for complete stabilization at a particular load In general, balanced stabilizers require occasional adjustment and this is an unsatisfactory feature in a commercial design. It is better practice to use a degenerative type designed with a degree of stabilization adequate for the purpose in view.

A comprehensive review of the subject was made by Hunt and Hickman[51] who give expressions for computing the performance of ten types of circuit, which they have classified into four groups :

(1) Transconductance Bridge
(2) Amplification Factor Bridge
(3) Simple Degenerative Amplifier
(4) Combination circuits involving two or more of the foregoing classes.

Useful discussions of the degenerative type have been given by a number of authors[56–59]. They give other references, and as the literature is rather extensive no further discussion will be made here of the many arrangements that have been developed.

The limiting factor in h.t. stabilizers which employ single-valve stages of amplification of the negative feedback voltages, is the loss of stabilization with changes in heater-supply voltages. In the present design this effect is reduced by a substantial factor, first by the use of coupled-cathode circuits throughout for the same reasons that they were employed in the early stages of signal amplification, and, secondly, the provision of stabilized direct-current supply to the heaters in the second and third stages of stabilization.

A " floating " type of supply must simulate the characteristics of the battery h.t. supply that it replaces, as already discussed in Section 2.1.3. The present circuit employs three stages of stabilization. The first is a series-type degenerative stabilizer which is situated on the power-unit chassis. The output voltage is 320 V and this supply is used for the third and fourth stages of the amplifier. The second stage of stabilization is situated physically on the amplifier chassis

adjacent to the input stages of the amplifier: the output is at 220 volts, with a ripple level not exceeding 15 μV: the negative pole is common to the other supply and is maintained at $-$ 70 volts to earth potential by a third stage of stabilization. The potentials to earth of the two positive supplies are consequently + 150 V and + 250 V.

The third stage of stabilization is, with the second, at the amplifier chassis and the two form a symmetrical pair of circuits. The "floating" supply feeds the second stage of the signal amplifier, which is part of a cascade circuit which includes the first stages of amplification and the cathode-coupling pentode load of Fig. 10(*a*). This special supply is equivalent to a carefully insulated high-tension battery. The function of the third stabilizer circuit is to degenerate the amplitude of any induced or leakage currents between the h.t. supply and chassis earth.

The reference voltage for the second and third stages of stabilization is derived from a cold-cathode valve, followed by an RC filter circuit. The first stage of stabilization is controlled by the output voltage of the second which is transmitted back to the power-unit chassis through a screened pair of wires cabled with the other interconnections. This cable has to be several feet in length to allow for opening the cabinet in which the chassis are mounted. Any voltage picked up electrically or electromagnetically appears equally in each conductor of the pair, and consequently it is rejected by the coupled-cathode amplifier employed in the first stabilizing stage.

In both the h.t. and the heater supply stabilizers, the cold-cathode valve which provides the reference voltage is followed by an RC filter with a time constant of several seconds (i.e., several times the constant for the signal amplifier), for the purpose of reducing the effects of noise and fluctuation in voltage that are always present in these valves. The filter reduces the rate of change of fluctuations in reference voltage below that rate to which the amplifier will respond. A cyclic fluctuation or "ticking" of up to 100 mV is commonly experienced with 70-volt regulating valves. Specimens exceeding this value are not used in this position in the circuit.

(To be concluded)

ELECTROMAGNETIC WAVES IN A VACUUM

Relative Directions of the Electric and Magnetic Vectors

AN interesting letter on this subject by Dr. N. S. Japolsky was published in *Nature* of April 26. He pointed out that the usual assumption that the electric and magnetic vectors are mutually perpendicular is only true in certain circumstances. The curl of H is perpendicular to the time derivative or rate of change of D, and not necessarily to the vector D, and similarly the curl of $\mathfrak{E}$ is perpendicular to the time derivative of B and not necessarily to the vector B itself.

In plane-polarized plane waves they are mutually perpendicular, but in rotating waves in which the vectors rotate, they are not mutually perpendicular except in some special cases, such as circularly-polarized plane-waves, in which the electric and magnetic vectors are perpendicular to each other and rotate with the same angular velocity. They are also mutually perpendicular in spherical waves as radiated from an ordinary aerial, the electric vector being always in the axial plane, and the magnetic vector perpendicular to it.

Except for such special cases, in waves in which the vectors rotate they are not mutually perpendicular. As Japolsky says, this point is usually overlooked because until quite recently very little attention was given to the study of such waves. Dr. Japolsky was at one time at the Technological Institute of Leningrad, but has been for many years at the Royal Institution in London. He has contributed several articles to the *Philosophical Magazine* dealing with what he calls electromagnetic whirls, the idea being that electronic particles may really be microscopic electromagnetic whirls, in which, as he points out, the electric and magnetic vectors may not be mutually perpendicular.

G. W. O. H.

NEW BOOKS

Television Simplified

By MILTON S. KIVER. Pp. 375 + vii, with 222 illustrations. Macmillan & Co., Ltd., St. Martin's Street, London, W.C.2. Price 27s.

This book is of American origin and all the examples are taken from American practice, which differs quite considerably from British television in certain important matters, such as, the use of negative modulation, horizontal polarization, vestigial sideband transmission and frequency modulation for the sound channel. In spite of this a very large amount of the material in the book is as applicable in this country as in the U.S.A.

In his preface the author says " It is the purpose of this volume to aid the thousands of radio men and women who will design, construct and repair television sets to bridge the gap between the modern ' pure ' sound receiver and the more complex television circuits." He has succeeded in his purpose and the book should form an excellent introduction to television for those who are well versed in the practice of sound equipment but have little knowledge of television.

The book starts off with chapters on the general principles of television, very short waves and television aerials, and then goes on to treat in more detail wide-band circuits, detectors, and a.g.c. circuits, v.f. amplifiers, d.c. restoration, c.r. tubes, synchronizing and deflexion systems. There is a chapter on complete receivers, and others on colour television, frequency modulation and servicing.

The treatment is almost entirely non-mathematical, but the explanations of the operation of the various circuits are clear and will unquestionably be helpful to the beginner. There are very few errors, but there are some. On p. 199, the author refers to a triode or pentode limiter fed with a v.f. signal having positive-going sync pulses and providing d.c. restoration in the grid circuit. The *circuit* is substantially the same as one previously shown as an amplifier and he comments that the reader may be puzzled to distinguish between the two. He goes on to say that the difference lies in the value of the grid leak which is 0.5 MΩ for d.c. restoration in an amplifier and 1–3 MΩ for a limiter. The difference of performance, however, occurs not through the use of a different value of grid leak, but because a much lower anode voltage (triode) or screen voltage (pentode) is used for a limiter than for an amplifier.

Something has also gone wrong with Fig. 9.12 which shows the sync pulse waveforms for even and odd frames. The line pulses are drawn coincident prior to the frame pulse, but displaced after the frame pulse.

The deflexion circuits and their characteristics are dealt with in a very superficial manner and while this may seem adequate for the beginner there is nothing to warn him of the many difficulties and pit-falls encountered in practice. W. T. C.

Interference from Industrial R.F. Heating

By A. TURNEY, B.Sc. Technical Report M/T 88. Published by The British Electrical & Allied Industries Research Association, 15, Savoy Street, London, W.C.2. Price 3s. 6d.

Elementary Radio Servicing

By WILLIAM R. WELLMAN. Pp. 260 + xi. Macmillan & Co. Ltd., St. Martins St., London, W.C.2. Price 21s.

Plastics for Electrical and Radio Engineers (2nd Edition)

By WALTER J. TUCKER and R. S. ROBERTS. Pp. 167 + xi. The Technical Press Ltd., Gloucester Road, Kingston Hill, Surrey. Price 15s.

British Research in the Radio Field

From the Secretary, The Institution of Electrical Engineers, Savoy Place, Victoria Embankment, London, W.C.2. Price 1s.

A report from the Research Committee of the Institution which reviews the facilities for research in the radio field and which makes recommendations for the stimulation and co-ordination of British radio research.

Wireless Direction Finding (4th Edition)

By R. KEEN. Pp. 1059 + xii with 630 illustrations. Iliffe & Sons Ltd., Dorset House, Stamford Street, London, S.E.1. Price 45s.

The B.B.C. Year Book, 1947

The twentieth B.B.C. Year Book, giving a review of the year's broadcasting, including television.

Pp. 152. B.B.C. Publications Dept. The Grammar School, Scarle Road, Wembley, Middx. Price 2s. 6d.

Lewis's Medical, Scientific and Technical Lending Library (Supplementary Catalogue), 1944-1946.

Pp. 176. H. K. Lewis & Co., Ltd., 136, Gower St., London, W.C.1. Price 5s. to non-subscribers, 2s. 6d. to subscribers.

Television.

Vol. III (1938–1941), Pp. 486 + xii; Vol. IV (1942–1946), Pp. 510 + xiv.

Published by R. C. A. Review, Radio Corporation of America, R.C.A. Laboratories Division, Princeton, New Jersey, U.S.A. Price $2.50 each, or $1.50 each, paper bound.

Collection of papers by R.C.A. authors. Many are reprinted in full, but some are summarized. Appendices to Vol. III include summaries of the papers of Vols. I and II which are out of print and which cover 1936–1938.

Electronic Engineering Patent Index

Edited by FRANK A. PETRALGIA. Pp. 476 + viii. Published by Electronics Research Publishing Co., 2, West 46th St., New York 19, N.Y., U.S.A. Price $14.50.

Contains some 2,000 patents granted during 1946, classified according to subject and reproduced in full from the annual file for 1946 of the Gazette of the U.S. Patent Office.

Rubber in Industry

" Developments in Rubber " is a loose-leaf binder containing a number of reprints of articles dealing with the technical aspects of rubber and rubber-metal bonds in industry. Copies are available for Engineers and Education Institutions and applications should be made on official notepaper to Andre Rubber Co., Ltd., Kingston By-Pass, Surbiton, Surrey.

CORRESPONDENCE

Letters of technical interest are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

Comparison of A.M. and F.M.

To the Editor, " Wireless Engineer."

SIR,—We are accustomed to Britain, with smaller technical resources, being later than the U.S.A. in some branches of radio development, but even in this country we are not in the state of mind implied by M. G. Nicholson's paper in your July issue. F.M. has been studied here from the point of view of point-to-point communications even more than broadcasting, and communications engineers are well aware of the effects of bandwidth on fidelity and of the difference in atmospheric noise level between 1 Mc/s and 50 Mc/s and upwards.

In discussing adjacent-channel interference, Nicholson omits to explain that Hans Roder's paper, which was published in 1937 (only one year after Armstrong's classic f.m. paper), was concerned with the case in which the two interfering signals are allocated channels which overlap by 50 per cent ; i.e., one carrier is situated on the limiting frequency of the other channel. In the light of present-day knowledge this is an utterly unpractical condition, since two transmitters which are expected to be within usable range of the same receiving station would always be given clear channels.

I am surprised to hear that the suppression of a weak modulated signal by a strong carrier in a linear detector, the " rectifier discrimination " effect, " does not appear to be generally appreciated." For example, my paper on " F.M. Communication Systems " (*Wireless Engineer*, May, 1943), specifically mentioned this factor when discussing the effects of modulated interference in f.m. and a.m. systems. However, I overlooked at that time a point which Nicholson does not seem to have considered : in the usual f.m. receiver having a discriminator circuit and two diodes, the signal is converted from f.m. to combined f.m. and a.m. and the a.m. is then detected in linear rectifiers. Therefore, the " rectifier discrimination " phenomenon should still be effective in the f.m. receiver, in addition to the superior selectivity obtained in f.m. from the limiter and balanced discriminator.

We are familiar with the characteristics of pre-emphasis plus de-emphasis systems, and the figures quoted in my 1943 paper included an allowance for the necessary reduction in modulation depth (taken from the R.C.A. experimental results, Nicholson's Reference 5).

Since 1943, I have made some studies of impulse noise, but it must first be pointed out that Nicholson's radar analogy is fallacious. The discrimination between transmitted pulse and echo in a radar receiver depends entirely on the time difference, and the only amplitude requirement in the receiver is to avoid " paralysis " so that the time discrimination can be effective. A radar transmitter causes severe interference on an a.m. telephony receiver. As an alternative to f.m. for impulse elimination, wide-band a.m. plus an amplitude-limiter is proposed, but there are several objections to this :—

(i) The a.m. system must have the same bandwidth, and therefore the same transmitter channel spacing, as the f.m. system. But owing to the lack of " capture effect " on a.m., common-channel transmitters must be separated by much greater distances, and the total number of transmitters which can be operated is reduced.

(ii) The receiver must remain wide-band (e.g., 150 kc/s) up to the impulse-limiting circuit, and this may be embarrassing since the limiter should work at a high level to render negligible the transition region between conduction and non-conduction in the diode.

(iii) The impulse limiter must be a fairly elaborate circuit in order to follow the modulation envelope, and if clipping of the higher modulation frequencies is to be avoided, only those components of the pulse which are substantially higher in frequency can be cut.

The net result is that the wide-band a.m. system can never be quite as good as a correctly adjusted f.m. system (which cuts impulses exactly at modulation level), is about as costly as the additional i.f. limiter stage in the f.m. receiver, and lacks the f.m. advantages on all other types of noise.

So far as I am aware, all properly controlled tests of f.m. versus a.m. systems have led to the conclusion that the balance of advantage is with f.m. This would appear to be supported by the decision of the B.B.C., after the extensive trials reported in the *B.B.C. Quarterly* of July, 1946, to continue with f.m. development, and by the use of f.m. for multi-channel telephone links by the British Post Office.

D. A. BELL.
British Telecommunications Research Ltd.,
Taplow, Bucks.

" The Doppler Effect in Propagation "

To the Editor, " Wireless Engineer."

SIR,—Your correspondents, Messrs. R. E. Burgess and F. S. Atiya did quite right to point out in the August issue the error in my article " Doppler Effect in Propagation," wherein the implications of relativity principles were neglected. In the interest of demonstrating the approximate relationship between the observed " shift " of the Washington standard frequency and the changes in propagation geometry, the fallacy of the two " classical " Doppler equations was unobserved. Fortunately, as Burgess agrees, the error is negligible in its effect on the remainder of the argument. I can confirm that at least one quite well-known and recently published text-book of Optical Physics commits the same error as I did.

As Mr. Essen says in his letter, the limitation of accuracy imposed by the Doppler Effect upon the comparisons between frequency standards remote from one another, using a high-frequency radio link, was demonstrated as long ago as 1935. Improvements in the accuracy with which the

absolute value of frequency standards can be determined and the greater knowledge of propagation conditions now available, suggested that the subject of the article might yet be of interest.

Replying to Mr. Essen, the average value of the apparent frequency-shift during steady, daytime conditions over several days at 1600 G.M.T. was chosen because the time was then about the most suitable for the highest-frequency reception with a relatively stable path. It was also a convenient time because the comparisons apparatus was not in use for other work. The example chosen was then a shift of -3 parts in 10^8. Fairly steady beats of as great as ± 6 parts in 10^8 have at various other times been obtained. Much greater, but widely varying, difference frequencies have been indicated during disturbed or rapidly changing conditions of reception, complicated by wave-interference associated with "scattering" and with multiple-path signals, one or another path being only momentarily predominant.

Differential propagation of high and low frequency transmissions from WWV has also been observed. For example, some comparisons showed frequency errors which themselves diverged by 4 parts in 10^8, with fairly steady beats with the local standard in both cases, sometimes with a simultaneous difference in sign. This effect does not seem to persist for longer than 15 or 30 minutes and is difficult to demonstrate, because it requires comparatively stable conditions on two frequency bands undergoing divergent treatment in the ionosphere, with freedom from random scattering, etc., in both bands.

H. V. GRIFFITHS.

Westerham,
Kent.

Cavity Resonators and Electron Beams

To the Editor, "The Wireless Engineer"

SIR,—In a recent article on "Cavity Resonators and Electron Beams," Mr. J. H. Owen Harries has derived some results which seem to show that there are fundamental limitations to the upper frequency which can be generated by beam-resonator combinations. Furthermore, the upper frequency appears to be of the order of 30,000 Mc/s. Of course, Mr. Harries' results do not forecast a sharp cutoff frequency but they do show a very marked falling off in efficiency above this frequency.

These results appeared to contradict the experimental evidence which shows that the efficiencies of both magnetrons and reflex klystrons do not alter very rapidly with frequency from 1,000 Mc/s to the highest frequencies yet reached.

An examination of Mr. Harries paper shows that his results do not give the correct frequency dependence of efficiency. In his work the efficiency of the circuit (the proportion of the energy generated which can be coupled into a load) is given by an expression of the form

$$\eta_c = 1 - b\left(\frac{\lambda_0}{\lambda}\right)^{\frac{8}{3}} \quad \ldots \quad \ldots \quad \ldots \quad \text{Equ. (38)}$$

where b is a constant for a given tube geometry and type of modulation. λ_0 is a reference wavelength taken as 10 cm in his work and λ is the wavelength at which η_c is to be measured. The correct expression (under the same approximations as Mr. Harries) is

$$\eta_c = 1 - b\left(\frac{\lambda_0}{\lambda}\right)^{\frac{1}{3}}$$

Mr. Harries Equ. (39) for the output power should read

$$P_L = \frac{V_0 B^2 \eta_0}{R_{B0}}\left[1 - \frac{0.77}{\eta_0}\frac{1}{\Psi_0}\left(\frac{\lambda_0}{\lambda}\right)^{\frac{1}{3}}\right]$$

The reasoning which leads to these changes is as follows. Harries parameter Ψ can be shown, after reduction, to equal R_0/R_b where R_0 = parallel resonant impedance of resonator and R_b = beam impedance.

Equ. (28.2) gives $\eta_c = 1 - b/\Psi$

so we have to investigate the frequency dependence of R_0/R_b. It is well known that $R_0 \propto \lambda^{\frac{1}{3}}$ for similar resonators; Harries states that $R_b \propto \lambda^{-2}$ but this is not correct. There are two cases to be considered (a) Electrostatic focusing, and (b) Magnetic focusing.

Case (a): Maximum current through a tunnel length l, diameter $d = 38.9 \times 10^{-6}\left(\frac{d}{l}\right)^2 V^{\frac{3}{2}}$ amps.

For similar resonators this is invariant; R_b is therefore a constant for a given resonator shape.

Case (b): In the magnetic case the maximum current is given by $32.5 \times 10^{-6} V^{\frac{3}{2}}$ which is independent of the hole area or length. R_b is again a constant.

We therefore see that $R_0/R_b \propto \lambda^{\frac{1}{3}}$ and not as $\lambda^{\frac{8}{3}}$ which leads to the results given above. It should perhaps be pointed out that a much more refined analysis, dealing with the actual process of energy conversion in any given type of tube is necessary before the results can be brought into line with experiment. Experiment does, however, agree approximately with the $\lambda^{\frac{1}{3}}$ variation.

It appears that, though no tube engineer is likely to under-estimate the practical difficulties, there are no fundamental limitations to the generation of power at millimetre wavelengths. It also seems unnecessary to postulate a connection between the difficulties of millimetre generation and the various new phenomena in the field of propagation research.

A. H. BECK.

Standard Telecommunication
Laboratories, Enfield, Middx.

To the Editor, "Wireless Engineer"

SIR,—Mr. Beck bases his criticism upon the deduction that R_b is a "constant"; that is, that the total electron beam current that can enter the resonator from a cathode is independent of the area of the cathode and of the area of the aperture through which the electrons enter the resonator. This is the kind of deduction which Euclid called "absurd"! Mr. Beck quotes his cases (*a*) and (*b*) in support, but his formulae are not complete expressions for the electron beam parameter R_b. For example, no account is taken of the cathode-source of the electrons.

His argument therefore falls to the ground. I do not know to what experiments Mr. Beck refers when he says that they contradict my results.

Mr. Beck misquotes me at the extreme end of his

letter. I did not postulate a " connection " between the propagation difficulties found around 1 cm wavelength and the existence of a limit to the performance of resonator-valves at the same part of the wavelength gamut. On the contrary, I said (Section 10) that it was a " singular coincidence " that two unconnected but fundamental phenomena should both adversely affect radio communication at the same part of the gamut. I am sorry that this does not seem as interesting to Mr. Beck as it does to me, but I am, I fear, unrepentant and still find it well worthy of comment that, at this particular part of the gamut, one finds not merely one but two separate obstacles which have to be taken into account when one is trying to establish communication.

J. H. Owen Harries.

Harries Thermionics Ltd.,
London, W.1.

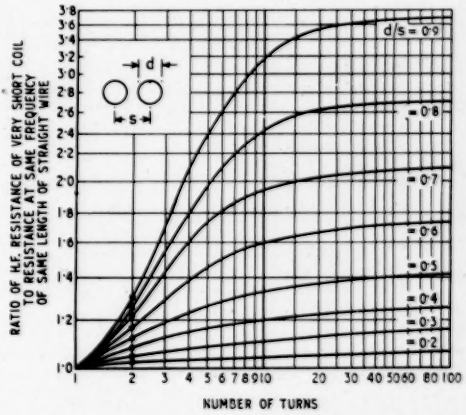
Q of Solenoid Coils

To The Editor, " Wireless Engineer "

Sir,—Mr. Callendar points out (*Wireless Engineer*, June 1947) that there is little indication in my article as to how far Butterworth's correction factor for coils of few turns can be relied on. That is a deficiency of which I was acutely aware and I can only plead lack of time for making the large number of measurements necessary to fill this gap.

Very tentatively, we can deal with short coils in the following way. Suppose we try to plot curves of ϕ (ratio of h.f. coil resistance to resistance of the same length of straight wire at the same frequency) against the number of turns (n), for various spacing ratios. We know that the curve has to approach asymptotically the values given in column 1 of Table VIII (*Wireless Engineer*, March 1947, p. 88). Also we know two more points, those for $n = 1$ and $n = 2$ values come from Butterworth's exact solution of the problem of two parallel wires carrying high-frequency currents in the same direction (*Proc. Roy. Soc.*, 1925, 107A, p. 708). From these we can draw a plausible looking curve, being guided by the very rough device suggested previously (*Wireless Engineer*, February 1947, p. 39) namely that ϕ should be diminished by $\frac{100}{n}\%$ when n is greater than, say, 20. The results of this procedure are shown in the diagram. Of course, as the ratio of length to diameter increases the curves become modified even for the smaller n values, in what is at the moment, an unpredictable way.

All this juggling with doubtful approximation is clearly quite unsatisfactory, and, as Mr. Callendar



remarks, further experimental work would be useful. It has, however, to be remembered that exact knowledge of the h.f. resistance of coils of two or three turns is likely to be required rather infrequently. The total resistance of circuits containing such coils will usually be dominated by other features of the circuits.

R. G. Medhurst.

G.E.C. Research Laboratories,
Wembley.

WIRELESS PATENTS

A Summary of Recently Accepted Specifications

The following abstracts are prepared, with the permission of the Controller of H.M. Stationery Office, from Specifications obtainable at the Patent Office, 25, Southampton Buildings, London, W.C.2, price 1/- each.

DIRECTIONAL AND NAVIGATIONAL SYSTEMS

581 683.—Amplifier utilizing selective feed-back through a delay network, particularly for the pulsed signals employed in radiolocation.

Standard Telephones and Cables Ltd. and M. M. Levy. Application date 6th June, 1941.

581 696.—Radiolocation system in which the exploring beam covers a conical field of observation in alternately diverging and converging spiral paths.

A. L. Hodgkin. Application date 16th September, 1943.

581 724.—Directive-aerial system including a horn-shaped reflector having two variable earthing points for oscillating the maximum lobe of radiation in order to define an approach path.

Standard Telephones and Cables Ltd. and E. O. Willoughby. Application date 2nd May, 1941.

581 762.—Directive aerial comprising a number of forked waveguide dipoles, each provided with spaced apertures, for controlling the relative phasing of the energy-flow inside and outside the guides.

Board of Trustees of the Leland Stanford Junior University. Convention date (U.S.A.) 10th July, 1940.

581 806.—Radiolocation system in which a cathode-

ray indication is given only when echo-signals of equal strength are received on each of two diverging d.f. aerials.

C. S. Agate and A. H. Cooper. Application date 25th September, 1941.

582 933.—Radiolocation indicating-device in which a neon flash-lamp co-operates with a compass-scale rotating synchronously with the exploring beam.

R. J. Stevens. Application date 16th April, 1941.

582 934.—Preventing variable-polarization effects in the transmitted and reflected signals of a radio-location system.

Marconi's W.T. Co. Ltd., J. M. Furnival, N. M. Rust and G. E. Partington. Application date 5th August, 1941.

583 036.—Receiver for radiolocation in which, during each quiescent interval, the gain is progressively increased whilst the bandwidth is progressively decreased.

The British Thomson-Houston Co., Ltd (communicated by the General Electric Co.) Application date 30th September, 1943.

583 320.—Balanced-bridge arrangement for minimizing undesired pick-up by the horizontal leads of an Adcock or like aerial.

P. G. Redgment and C. S. Wright. Application date 13th September, 1944.

583 361.—Radio-beacon of the equi-signal type in which the two transmitting aerials are energized in fixed out-of-phase relation, and distinctive "on" and "off" course signals are produced by periodically interrupting the radiation from at least one aerial.

W. J. O'Brien. Convention date (U.S.A.) 2nd March, 1942.

583 475.—Radiolocation system in which the echo-signal is brought into line with a datum calibration mark against a time-base which can be speeded-up to permit the closer scrutiny of a selected signal.

C. A. Lewis, R. J. Pumphrey, O. L. Ratsey and D. S. Watson. Application date 27th August, 1943.

583 743.—Aerial switching arrangement, particularly for radiolocation, in which quarter-wave blocking stubs are opened and closed by spark-gap discharges.

C. J. Banwell, C. H. Westcott and R. J. Lees. Application date 24th November, 1944.

RECEIVING CIRCUITS AND APPARATUS

(See also under Television)

583 285.—Method of wiring-up the component parts, say of a radio receiver, by depositing a metallic-powder paste upon a carrier or chassis of insulating material.

Murrayfield Nominees Ltd. and P. Plowman. Application date 13th October, 1944.

583 558.—Superheterodyne mixing-stage having coaxial-line resonators for handling frequencies that are too high for direct amplification.

Standard Telephones and Cables Ltd. (assignees of G. J. Lehmann). Convention date (U.S.A.) 30th October, 1943.

583 800.—Diode shunt across the input circuit of a detector valve to limit the effect of interference.

M. Slaffer and C. S. Wright. Application date 19th December, 1942.

584 014.—Motor-car radio set in which the volume and tuning controls, as well as the on-off switch, can be set for operation by the foot of the driver.

E. F. McDonald, Jr. Application date 3rd July, 1944.

TELEVISION CIRCUITS AND APPARATUS

FOR TRANSMISSION AND RECEPTION

583 183.—Preparing and processing the photo-sensitive mosaic electrode of a television transmitter c.r. tube.

Co. Para la Fabricacion de Contadores y Material Industrial and P. Viteau. Application date 15th February, 1944.

TRANSMITTING CIRCUITS AND APPARATUS

(See also under Television)

583 136.—Two-scale instrument for promptly indicating when a radio transmitter is over-modulated.

A. E. Hochstein. Application date 4th September, 1944.

583 161.—Balanced bridge made of transmission-line elements for protecting the receiving circuits against outgoing signals in a combined transmitter and receiver.

The General Electric Co. Ltd. and D. C. Espley. Application date 2nd January, 1941.

583 465.—Construction and operation of an adjustable plunger or shorting-piece used for tuning transmission-line circuits.

T. C. Finnimore and C. S. Wright. Application date 1st July, 1942.

583 697.—Construction of waveguide having sufficient longitudinal flexibility to permit some degree of bending without loss of electrical efficiency.

Callender's Cable and Construction Co. Ltd. and D. T. Hollingsworth. Application date, 17th November, 1944.

583 951.—Junction-piece for coupling a single waveguide to a forked or divided waveguide.

G. E. F. Fertel and C. S. Wright. Application date 7th June. 1944.

SIGNALLING SYSTEMS OF DISTINCTIVE TYPE

582 979.—Pulse-modulation system in which two saw-tooth oscillation-generators are fed with out-of-phase components of a common frequency.

Standard Telephone and Cables Ltd. and R. B. Shepherd. Application date 11th September, 1942.

583 262.—Multivibrator gating-circuit for selecting a given pulse-train from other trains of different repetition-frequency, particularly for multichannel signalling systems.

Standard Telephones and Cables Ltd. and C. W Earp. Application date 26th November, 1943.

583 511.—Double-stability pulse-modulating system in which the leading and trailing edges of each square waveform are significant factors.

Standard Telephones and Cables Ltd. and D. M. Ambrose. Application date 26th September, 1941.

583 799.—Duration-modulated pulsing system in which the modulation-depth is controlled so as to economize power.

Standard Telephones and Cables Ltd., P. K. Chatterjea and C. T. Scully. Application date 7th August, 1942.